MULTI-SENSOR DATA FUSION FOR LAND VEHICLE ATTITUDE ESTIMATION USING A FUZZY EXPERT SYSTEM

Jau-Hsiung Wang* and Yang Gao

Department of Geomatics Engineering, The University of Calgary, 2500 University Drive NW, Calgary, Alberta, Canada T2N 1N4
Email:wangjh@ucalgary.ca and gao@geomatics.ucalgary.ca

ABSTRACT

In Inertial Navigation Systems (INS), the attitude estimated from gyro measurements by the Kalman filter is subject to an unbound error growth during the stand-alone mode, especially for land vehicle applications using low-cost sensors. To improve the attitude estimation of a land vehicle, this paper applies a fuzzy expert system to assist in multi-sensor data fusion from MEMS accelerometers, MEMS gyroscopes and a digital compass based on their complementary motion detection characteristics. Field test results have shown that drift-free and smooth attitude estimation can be achieved and will lead to a significant performance improvement for velocity and position estimation.

Keywords: Data Fusion, Expert System, Fuzzy Logic, INS, MEMS

1. INTRODUCTION

Global Positioning Systems (GPS) have found wide applications in land vehicle navigation for its cost-effectiveness and long-term accuracy (Parkinson & Spilker, 1996). But its reliability and availability will be significantly degraded in the presence of signal blockages, interference and multipaths, especially in urban area. A popular solution to this problem is the integration of GPS and INS to take advantage of their complementary characteristics. For land vehicle navigation, the INS sensors are low-cost and small size MEMS-based inertial sensors whose instrument bias, drift and noise are significant.

Based on INS mechanization, the error in velocity and position estimation will mainly be governed by the accuracy of the estimated attitude (Titterton & Weston, 1997). In traditional approaches, only gyroscopes are used for attitude determination and the attitude errors are compensated for by the Kalman filter. Since Kalman filter is model-dependent, the system model parameters need to be precisely known a priori (Brown & Hwang, 1997). For low-cost MEMS sensors, precise knowledge (or modeling) of their significant instrument bias, drift and noise is very difficult in practical applications and subsequently this will affect the performance of the Kalman filter for attitude estimation. In particular, with only low-cost gyroscopes, the Kalman filter estimation errors will accumulate over time when there are no measurement updates, resulting in unreliable attitude solutions over a long prediction period (Brown & Lu, 2004).

In this paper, three types of low-cost sensors, namely MEMS accelerometers, MEMS gyroscopes and a magnetometer, are investigated for attitude estimation. A magnetometer with complementary characteristics to a gyroscope can provide absolute heading information relative to the magnetic north without time-accumulated errors. For tilt sensing, when a vehicle is static, only the accelerometer measurement that contains the gravity field can directly derive pitch and roll angles without time-accumulated errors. Since the physical characteristics of each sensor are related to vehicle dynamics, a fuzzy expert system is designed to identify the vehicle's motion. Once the motion type of the vehicle is identified, the most suitable sensor can be used to improve attitude estimation and to control the estimation error. Field tests using a van driven on a road were performed to examine the attitude accuracy estimated by the proposed system. The test results have shown that the proposed system can bound the attitude errors and reduce the error growth if vehicle stop is available. The performance improvement in velocity and position determination using the fused attitude is also discussed.

2. ATTITUDE ESTIMATION BY MULTI-SENSORS

The principle of inertial navigation is to derive the attitude, velocity and position of a moving body by measuring its dynamics based on Newton's Law. To sense the dynamics of the vehicle, the INS is aligned with the body frame consisting of three orthogonal axes where x is in the direction of forward motion of the vehicle, z is in the down direction, and y is in the direction of transverse motion of the vehicle, perpendicular to the plane formed by x and z axes. In land vehicle navigation, the motion of a vehicle on the earth's surface is mostly represented in the navigation frame whose axes are aligned to the local east (e), north (n) and down (d). The transformation between the navigation frame and the body frame can be accomplished by a sequence of elementary rotations about the attitude angles. Therefore, the vehicle velocity and position in the navigation frame can be obtained when the vehicle attitude and the acceleration measured in the body frame are determined.

The attitude of the vehicle is represented by three Euler angles, roll (ϕ), pitch (θ), and yaw (ψ), which are the rotation angles about the x, y and z axes, respectively. The changes of Euler angles, called Euler rates, are relative to the rotation rates of the body frame which can be measured by gyroscopes directly in the following manner:

$$\dot{\phi} = \omega_{Bx} + \sin\phi \tan\theta \omega_{By} + \cos\phi \tan\theta \omega_{Bz} \tag{1}$$

$$\dot{\theta} = \cos\phi \omega_{By} - \sin\phi \omega_{Bz} \tag{2}$$

$$\dot{\psi} = \frac{\sin\phi}{\cos\theta}\omega_{By} + \frac{\cos\phi}{\cos\theta}\omega_{Bz} \tag{3}$$

where ω_{Bx}, ω_{By}, and ω_{Bz} are angular velocity of the body frame measured by gyroscopes.

The shortcoming of using gyroscopes to estimate attitude is the error accumulation due to the integration process. Small gyro biases will result in substantial error growths for unbound attitude. Especially with low-cost sensors, the attitude estimation would become unreliable at fast speeds since sensor errors are dynamic and difficult to model.

In contrast to gyroscopes, accelerometers can be used to directly derive vehicle pitch and roll angles while the vehicle is static or moving linearly at a constant speed. Under this condition the accelerometer output, which contains only the local gravity field can be used to determine the vehicle's pitch and roll angles as follows:

$$\phi = -\sin^{-1}\left(\frac{A_{By}}{g}\right) \tag{4}$$

$$\theta = \sin^{-1}\left(\frac{A_{Bx}}{g}\right) \tag{5}$$

where A_{Bx} and A_{By} are the acceleration of the body frame measured by accelerometers and g is the local gravity field.

According to Eqs. (4) and (5), no integration is required and therefore the tilt estimation error will not increase with time. The accuracy of the tilt estimation is mainly governed by the accelerometer's bias. Since the accelerometer's bias can be estimated by stationary leveling and its effect is diminished by the gravity field, the accelerometer-based tile estimation is more accurate than gyro-based estimation. Thus, accelerometers can be used to bound and reset the tilt information calculated by the gyroscopes when the vehicle is static or moving linearly at a constant speed. (Ojeda & Borenstein, 2002).

For a vehicle's heading determination, a magnetometer is able to provide absolute heading information relative to the magnetic north without time-accumulated errors (Caruso, 1997). But the compass measurements are still subject to the influence of nearby ferrous effects and interference. In land vehicle application, the nearby ferrous effects are mainly generated by the vehicle itself. They will remain stable if the compass is securely and properly mounted in a vehicle. On the other hand, if the interference is the result of magnetic disturbances from such things as DC currents, it will change over time randomly. In addition to these environmental magnetic effects, the declination angles must be determined to correct for true north. By properly setting up the compass in a land vehicle to minimize the nearby ferrous effects, we could approximately model the remained nearby ferrous effects and declination angles as the combination of the bias and scale factor error in the heading domain as follows.

$$\psi = \hat{\psi} + b_\psi + S_\psi \hat{\psi} + n_\psi \tag{6}$$

where ψ is the true heading, $\hat{\psi}$ is the heading provided by magnetometer, b_ψ is the sensor bias, S_ψ is the scale factor, and n_ψ is the noise and disturbance. If sufficient measurement and true value data are available, the biases and scale factors can be estimated by using the least squares method (Wang, 2004).

It should be noticed that in land vehicle applications the magnetometer is not confined to a level plane most often and its tilt angles should be determined for heading corrections (Caruso, 1997). Since the tilt information is very difficult to be accurately estimated using low cost sensors when a vehicle is moving, we only apply tilt compensation when a vehicle is stationary. Thus, the magnetometer heading will be used to bound and reset the heading information calculated by the gyroscopes only when a vehicle is not moving.

Once the vehicle attitude is determined, the vehicle velocity and position in the navigation frame can be derived from accelerometer measurements based on the vehicle's dynamics model. In this paper we have applied the constrained motion model proposed by Brandt and Gardner (1998). In normal driving condition, the vehicle can be assumed to have no motion along the transverse direction and normal to the road surface. The vehicle motion constraints can be applied to simplify the mechanization equations and reduce the navigation errors. The constrained motion model is defined as follows (Brandt & Gardner, 1998):

$$\dot{V}_f = A_{Bx} - g \sin\theta \tag{7}$$

$$\dot{x}_t = V_f \cos\theta \cos\psi \tag{8}$$

$$\dot{y}_t = V_f \cos\theta \sin\psi \tag{9}$$

where V_f is the vehicle forward velocity. x_t and y_t are the vehicle coordinates in the XY plane of the earth-fixed tangent frame.

Based on Eqs. (7) to (9), the accuracy of the velocity and position solutions are mainly dominated by the pitch and heading errors. Thus, in this study we only assess the accuracy of the pitch and heading estimation results.

3. A FUZZY EXPERT SYSTEM FOR MULTI-SENSOR DATA FUSION

As mentioned in the previous section, the performance and characteristics of each sensor are related to the vehicle's dynamics. Based on a knowledge of the specific physical shortcomings and strengths of each sensor modality under different motion conditions, more accurate attitude estimation can be achieved by multi-sensor data fusion. Thus, the association between the raw measurements and the vehicle dynamics should be investigated. In this paper, we apply a fuzzy expert system for the identification of vehicle dynamics. Once the motion type of the vehicle is identified, the most suitable sensor can be used to estimate the vehicle's attitude. In the meantime, the errors of the unused sensors can also be estimated based on the statistical information of the observations. More specifically, we will use accelerometers and magnetometers to derive tilt and heading information and estimate gyro drift using the least squares method when a vehicle is static. When a vehicle is moving, we will use the compensated gyro measurements to estimate the vehicle's attitude. The block diagram of our fuzzy expert system is shown in Figure 1.

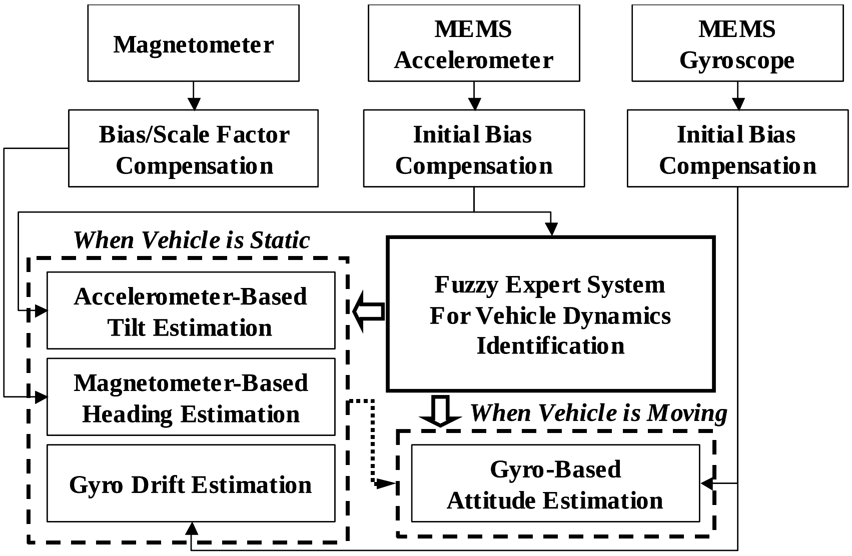

Figure 1. System Block Diagram

To correctly identify vehicle dynamics (static/moving) based on low-cost sensor measurements, the identification system must have the capacity of dealing with uncertainty and imprecision due to the noisy measurements and vehicle vibration effects. Both probability-based and fuzzy set theory based methods can handle the uncertainty and imprecision of data. However, the failure of the probability-based method in situations where little or no a priori information is known provides an arena for the use of a fuzzy expert system (Kandel, 1992). A fuzzy expert system is an expert system which incorporates fuzzy sets and/or fuzzy logic into its reasoning process and/or knowledge representation scheme. The fuzzy set theory provides a natural method for dealing with linguistic term which is a very effective knowledge representation format for imprecise and uncertain information (Kandel, 1992). Described in the following is the development of a fuzzy expert system for land vehicle dynamics identification.

Shown in Figure 2 is the architecture of the fuzzy logic-based vehicle dynamics identification system. In this research, the Mamdani type fuzzy inference system, which is considered as the most commonly seen fuzzy methodology, has been used (Mamdani & Assilian, 1975). Defined as the summation of jerk magnitude in x, y, and z axes accelerometer data over a specific period, the input variables for the system can interpret the degree of vehicle motion. The definition of the input variables is described as follows:

$$AJ_x = \sum_{i=k-d}^{k} \left| \left(Jerk_x \right)_i \right| \tag{10}$$

$$AJ_y = \sum_{i=k-d}^{k} \left| \left(Jerk_y \right)_i \right| \tag{11}$$

$$AJ_z = \sum_{i=k-d}^{k} \left| \left(Jerk_z \right)_i \right| \tag{12}$$

where the subscript k is the present measurement index and the subscript d indicates the amount of jerk magnitude to process.

The purpose of taking the sum of jerk magnitude is to dilute the vibration and noise effect on observations and make the difference between stop and move in the accumulated jerk more significant. It should be noted that the summation process would result in a delay of information representation. For the output of the fuzzy inference system, we define a numeric rating between 0.05 and 0.95 to describe the vehicle dynamics grade. A lower rating value indicates a higher likelihood for the vehicle being static.

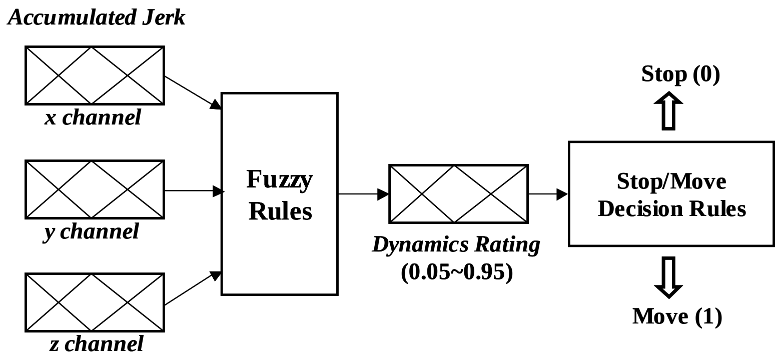

Figure 2: Fuzzy Logic-based Vehicle Dynamics Identification System

Once the inputs and output are defined for the system, the membership functions are further designed to define the quantity of the linguistic terms such as stop, uncertainty and move for fuzzy output. In this research, the design of the membership functions is based on our personal experience and knowledge gained from the field test data. At the same time, a set of rules is developed to describe the relationship between the input and the output. The rules established were essentially based on common sense reasoning and further modified through processing the field test data. The final tuned membership functions and rules are shown in Figure 3 and Table 1. Then, the output fuzzy set is converted into a crisp value using the center of the area method. It should be noticed that the fuzzy system design is vehicle dependent and sensitive to the location of the sensor installation.

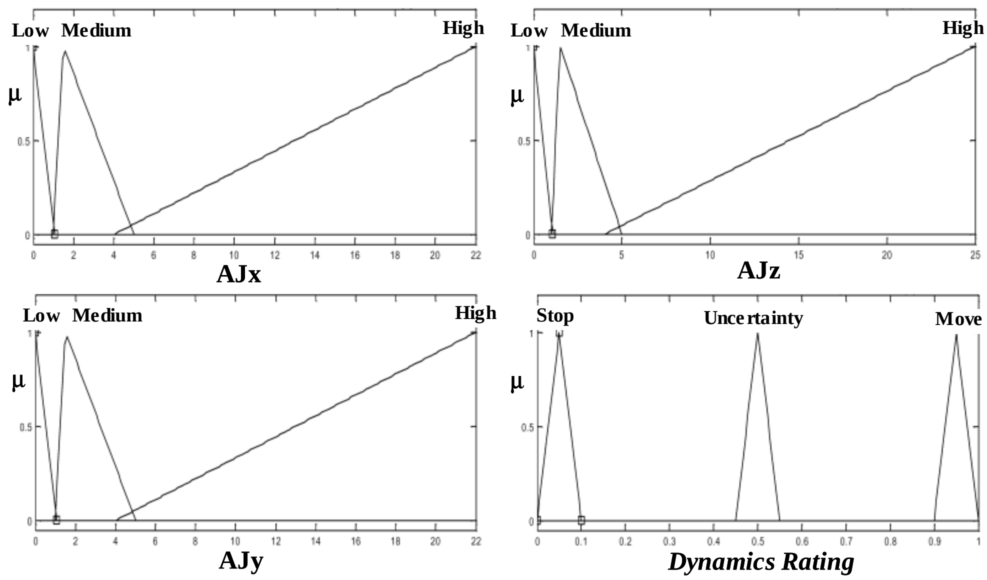

Figure 3. Membership Functions used in Fuzzy Expert System

Table 1. Rules used in Fuzzy Expert System

Rule No.	AJx	AJy	AJz	*Dynamics Rating*
1	High			Move
2		High		Move
3			High	Move
4	Medium	Medium	Medium	Move
5	Medium	Medium	Low	Uncertainty
6	Medium	Low	Medium	Uncertainty
7	Medium	Low	Low	Uncertainty
8	Low	Medium	Medium	Uncertainty
9	Low	Medium	Low	Uncertainty
10	Low	Low	Medium	Uncertainty
11	Low	Low	Low	Stop

The output of the fuzzy inference system represents the degree of motion of a vehicle. To correctly identify when a vehicle is at rest or moving based on the fuzzy output values, a set of decision making rules was designed as shown in Table 2. The rules 1, 2, and 3 work as a classifier to convert the continuous numeric rating values into a Boolean value to identify stopping or movement of a vehicle. Rule 4 is useful for instantly detecting the vehicle's movement and for avoiding the detection delay due to the use of the accumulated jerk as our fuzzy input.

Table 2. Rules used for Stop Identification

Rule 1	If *Dynamics Rating* equal 0.95, Then vehicle is moving.
Rule 2	If *Dynamics Rating* equal 0.05, Then vehicle is stop.
Rule 3	If *Dynamics Rating* is larger than 0.05 and smaller than 0.95, Then the present motion status follow the previous motion status.
Rule 4	If the present motion status is stop and the jerk magnitude in forward direction is larger than a criterion value, Then vehicle is moving.

As mentioned previously, we can use the accelerometers and the magnetometers to derive tilt and heading information and estimate gyro drift when the vehicle is at rest. Under these conditions, the vehicle's attitude would remain static; therefore, we can average the tilt and heading estimations to remove the noise effects. Comparing this static attitude information with the gyro-derived attitude, we can monitor the random walk of the gyro measurements and gyro bias effects on attitude estimation. In this research, we use least squares method to estimate the gyro noise and bias effects in attitude solutions which are the attitude drift error. The role of the least squares estimation is to optimally determine the attitude drift error in a statistical sense. The least squares problem can be described by the linear equation shown below.

$$L = AX \tag{13}$$

The observation, L, is the difference between the gyro-derived attitude at each epoch and its mean value in the stop periods. This value indicates the divergence of the gyro-derived attitude at each epoch. The design matrix, A, consists of the time difference from rest at each epoch. The unknown parameter, X, is the attitude drift error that needs to be estimated. Once the vehicle starts to move, we can use the observation and design matrix collected during the stationary periods to estimate the attitude drift error using the least squares method. Then, we can remove this drift error from the gyro measurements and perform gyro-based attitude estimation based on the Eq. (1) to (3). Because the roll angle is small and the rotation rate in z-axis is much larger than y-axis under general driving conditions, we can ignore the effects of y-axis rotation in the roll and yaw computation equation to reduce the errors from noise, vibration, and inaccurate measurements. For pitch rate estimation, however, both z-axis and y-axis rotation need to be used because the y-axis rotation isn't diminished by $\sin\phi$. It should be noted that while the vehicle isn't making any turn, the y-axis accelerometer measurement only contains the gravity field that can be used to derive the drift-free roll angle. For roll estimation during cornering, in this research we use the linear interpolation between the accelerometer-derived roll angle before and after turning to avoid the drift error caused by gyro-based estimation. However, the drawback of this method is the time delay in outputting the tilt estimation when the vehicle is cornering.

4. TEST RESULTS AND DISCUSSIONS

We performed several experiments to examine the performance of our proposed system. A low-cost MEMS-based inertial sensor, namely MT9 made by Xsens Inc., was used in the experiments. The MT9 is a digital inertial measurement unit that measures 3D rate-of-turn, acceleration and earth-magnetic field. The data output rate was chosen as 10 Hz. In the meantime, a carrier phase differential GPS (DGPS) solution with a data rate of 1 Hz was used to establish a reference for the position, velocity and heading. All of the sensors were mounted in a van and their outputs were logged and synchronized with the computer's timer for subsequent analysis. The test was performed in a parking lot at the University of Calgary. The reference trajectory provided by DGPS solution is shown in Figure 4. A few vehicle stops occurred during the test and the duration of the trip was about 6 minutes.

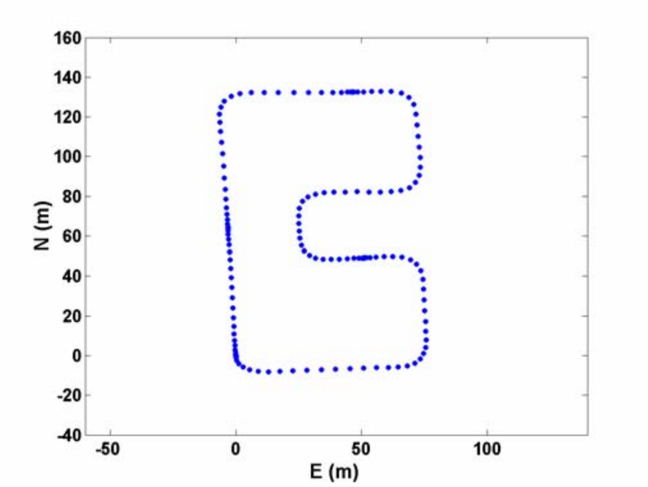

Figure 4. Test Trajectory

Figure 5 to 7 show the raw measurements of MT9 including the 3-axis acceleration, angular rate, and magnetic field of the body frame. It was shown that the accelerometer and magnetometer measurement are quite stable when the vehicle was at rest. The accelerometer measurement profiles also imply the diversity of vehicle jerk between rest and movement. For gyro measurements, the vehicle's rotation dynamics in the z-axis is much larger than the noise level. By contrast, the dynamics of pitching and rolling of a land vehicle is much lower than yawing while the gyro measurements in x-axis and y-axis are much more noisy due to the vehicle's vibration and road roughness.

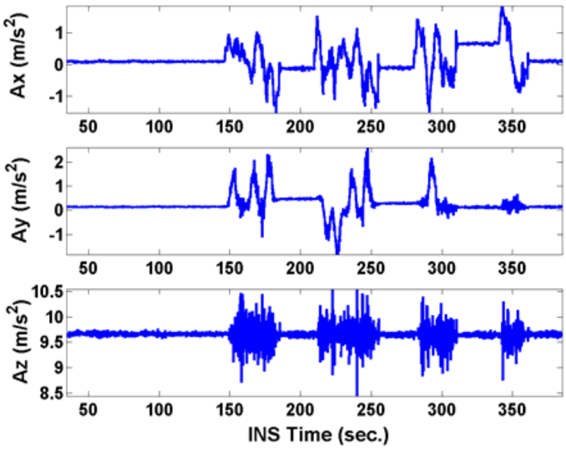

Figure 5. Raw Measurements - Accelerometer

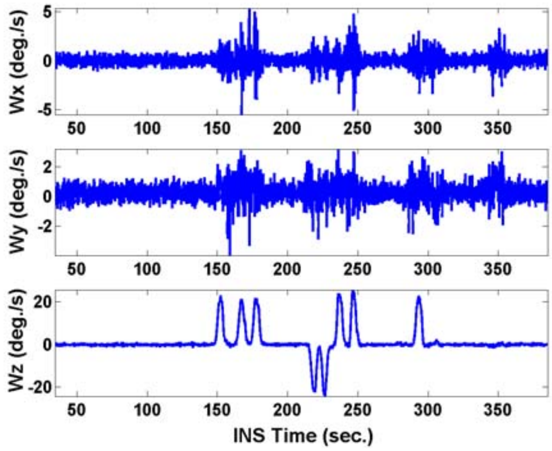

Figure 6. Raw Measurements - Gyroscope

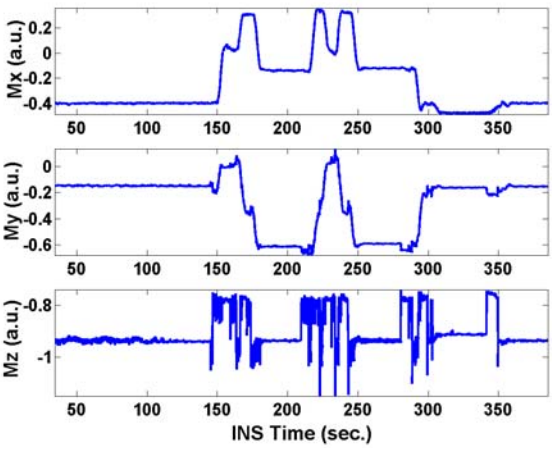

Figure 7. Raw Measurements - Magnetometer

Shown in Figure 8 is the result of vehicle dynamics identification provided by the proposed fuzzy expert system. The vehicle's stopping and movement have been correctly distinguished. The fuzzy expert system has properly interpreted the raw measurements and successfully recognized their relationship to the vehicle's dynamics.

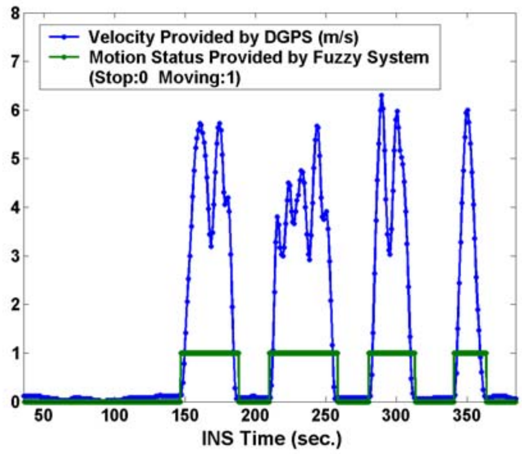

Figure 8. Vehicle Dynamics Identification

Figure 9 illustrates the heading angle derived from only gyro measurements (without aid) and modified by the fuzzy expert system, respectively. The reference heading is derived from DGPS velocity while the vehicle was in motion. When the vehicle was static, we can adopt the previous reference heading as the current reference. Obviously, the gyro drift errors have been controlled by the magnetometer-based heading update when the vehicle was at rest. On the other hand, when the vehicle was in motion, a smooth heading estimation that cannot be achieved by using a magnetometer because of the noise and tilt effects, has been accomplished by using the compensated gyro measurements.

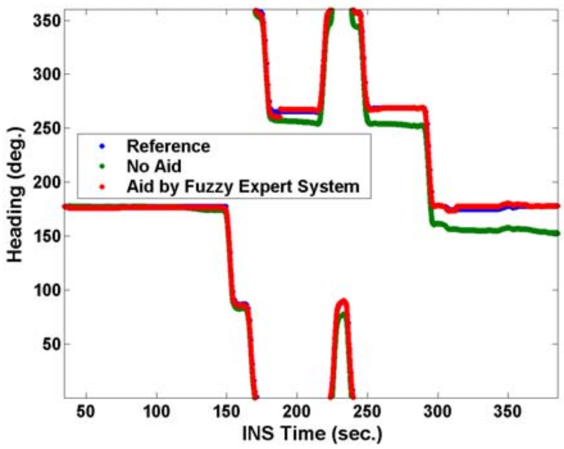

Figure 9. Heading Estimation

Figure 10 illustrates the pitch angle derived from only gyro measurements (without aid) and modified by the fuzzy expert system, respectively. Instead of the unaided pitch estimation with increasing error growth, the pitch estimation aided by the fuzzy expert system has been well bound and controlled. Since no reference pitch information was available in our test, we evaluated the performance of the pitch estimation by examining the velocity estimation calculated by Eq. (7). Figure 11 shows the velocity estimation using gyro-based (without aid) and data fusion-based (aid by a fuzzy expert system) pitch information, respectively. Obviously, the gyro-based data diverges quickly and cannot be used for navigation. In contrast, the velocity derived from the fuzzy expert system is very close to the reference velocity. Thus, the accuracy of the forward velocity estimation has been significantly improved by using the fusion-based pitch angle derived from the fuzzy expert system.

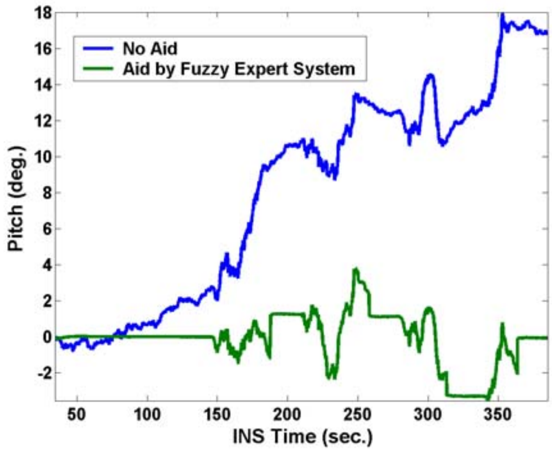

Figure 10. Pitch Estimation

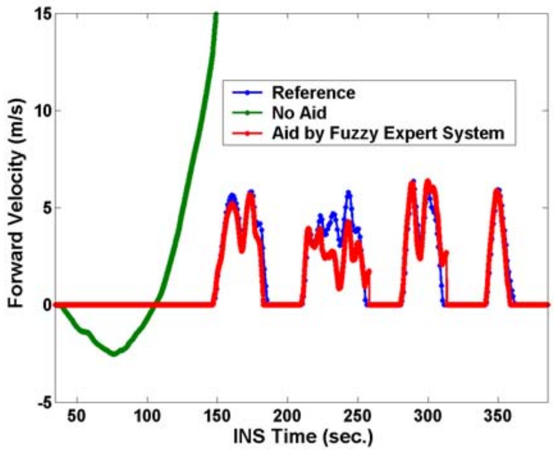

Figure 11. Velocity Estimation

To further assess the accuracy of heading, velocity and position solutions, we reduce the 10 Hz inertial navigation sampling states to 1 Hz and compare them with the synchronized DGPS data to examine the errors. Figure 12 shows the heading, velocity and 2D position estimation errors when we apply the proposed multi-sensor data fusion algorithm. Obviously, the heading and velocity errors have been well bound and controlled during this about 6-minute driving test (with a couple of stops in-between). In statistical analysis the mean and standard deviation (std) values of the heading error are –0.09 and 1.677 (degree), respectively. The mean and std values of the velocity error are –0.127 and 0.639 (m/s), respectively. In terms of the position solutions, the position error would accumulate with time due to the integration process. The final position error over this 6-minute stand-alone navigation period is about 50 meters, which is much better than the specifications supplied by the manufacture.

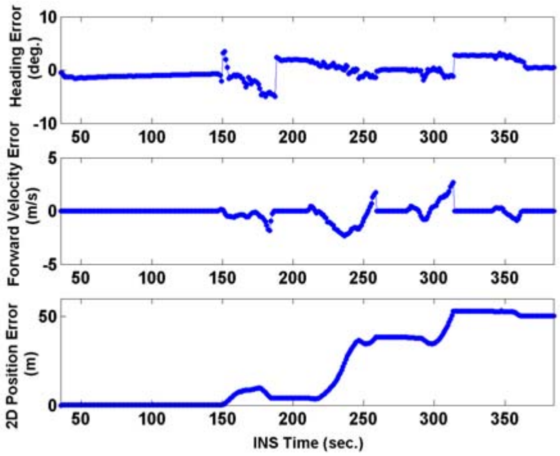

Figure 12. Heading, Velocity and Position Estimation Error

In order to more accurately assess the performance of the proposed system in the position domain, additional tests on the same route were repeated. The maximum and root mean square (RMS) values of the 2D position errors during the moving segment in each test are summarized in Table 3. It can be seen that there is a larger position error growth rate in the second segment and a smaller position error growth rate in the forth segment. This is because of the many instances of cornering motion (yawing motion) during the second segment but none during the forth segment. Based on Eq. (2), a small roll estimation error may result in a large error growth of the pitch estimates when yawing motion is significant. The test results also demonstrate that even in the same segment a certain variation in system performance exists. This is due to the bias variation and the large random walk of a low-cost sensor. In this paper, we applied the least squares method to estimate the gyro noise and bias effects in attitude estimation when the vehicle was stationary. When the vehicle starts to move, however, the noise is coupled with vibrations and sensor bias may change dynamically. Thus, the attitude drift errors may not be removed accurately and will cause position errors with different drift rate. In general, the proposed multi-sensor data fusion algorithm can provide bounded and smooth attitude estimation and further improve the navigation performance when a vehicle stops frequently.

Table 3. Statistical Analysis of 2D Position Error during Motion (a) Test 1 (b) Test 2 (c) Test 3 (d) Test 4

(a)

Moving Segment	2D Position Error		Periods (sec.)
	MAX (m)	RMS (m)	
1	9.84	6.23	41
2	34.82	21.23	49
3	20.05	6.67	33
4	5.01	3.02	23

(b)

Moving Segment	2D Position Error		Periods (sec.)
	MAX (m)	RMS (m)	
1	16.69	6.84	44
2	19.48	11.91	51
3	11.50	4.78	33
4	6.27	3.25	24

(c)

Moving Segment	2D Position Error MAX (m)	RMS (m)	Periods (sec.)
1	**19.47**	**12.68**	**42**
2	**43.40**	**13.12**	**49**
3	**19.84**	**10.78**	**31**
4	**8.49**	**5.02**	**21**

(d)

Moving Segment	2D Position Error MAX (m)	RMS (m)	Periods (sec.)
1	**29.98**	**14.46**	**40**
2	**45.29**	**25.64**	**36**
3	**33.85**	**12.78**	**24**
4	**5.48**	**3.64**	**15**

5. CONCLUSIONS

A new multi-sensor data fusion algorithm for land vehicle attitude estimation has been developed with the aid of a fuzzy expert system. First, we have investigated in-depth the physical characteristics of each low-cost sensor and its error sources related to vehicle motion. Then, a fuzzy expert system has been designed to correctly identify vehicle dynamics. Finally, based on the identified motion status, sensor error and attitude information was estimated by the optimal use of sensor modalities. The estimation in velocity and position using the fused attitude was also performed to explore the benefit of the proposed method for land vehicle navigation.

The results of the field tests have shown that the proposed method can provide adapted attitude estimation without unbound error drift and noisy disturbance. By using this fusion-based attitude, the accuracy of velocity and position estimation has been significantly improved. The proposed method can provide a desirable land vehicle navigation solution for one minute of stand-alone navigation using a low-cost MEMS-based inertial sensor when frequent stops are available. Further research to reduce error drift during motion caused by gyro bias variation, the large random walk of gyro measurements and cornering dynamics is recommended.

6. ACKNOWLEDGMENTS

This study was partially supported by the Natural Science and Engineering Research Council of Canada (NSERC) and the Network Centers of Excellence (NCE)'s Geomatics for Informed Decisions (GEOIDE) program.

7. REFERENCES

Brandt, A. & Gardner, J.F. (1998) Constrained Navigation Algorithms for Strapdown Inertial Navigation Systems with Reduced Set of Sensors. *Proceedings of the American Control Conference*. Philadelphia, Pennsylvania, USA.

Brown, A. & Lu, Y. (2004) Performance Test Results on an Integrated GPS/MEMS Inertial Navigation Package*. Proceedings of ION GNSS 2004*. Long Beach, CA, USA.

Brown, R.G. & Hwang, P.Y.C. (1997) *Introduction to Random Signals and Applied Kalman Filtering*. New York, NY: John Wiley & Sons.

Caruso, M.J. (1997) Application of Magnetoresistive Sensors in Navigation Systems. *Sensors and Actuators*. SAE SP-1220.

Kandel, A (Eds.) (1992) *Fuzzy Expert Systems*. Boca Raton, FL: CRC Press.

Mamdani, E.H. & Assilian, S. (1975) An Experiment in Linguist Synthesis with Fuzzy Logic Controller. *International Journal of Man-Machine Studies*, *7*(1), 1-13.

Ojeda, L. & Borenstein, J. (2002) FLEXnav: Fuzzy Logic Expert Rule-based Position Estimation for Mobile Robots on Rugged Terrain. *Proceedings of the 2002 IEEE International Conference on Robotics and Automation*. Washington DC, USA.

Parkinson, B.W. & Spilker, J.J. (Eds) (1996) *Global Positioning System: Theory & Applications Vol. 1, Progress in Astronautics and Aeronautics.* Washington, D.C.: American Institute of Aeronautics and Astronautics.

Titterton, D.H. & Weston, J. L. (1997) *Strapdown Inertial Navigation Technology*. London, UK: Peter Peregrinus.

Wang, J-H. (2004) The Aiding of a Low-Cost MEMS INS for Land Vehicle Navigation Using Fuzzy Logic Expert System. *Proceedings of ION GNSS 2004*. Long Beach, CA, USA.