ON SHRINKAGE ESTIMATION FOR THE SCALE PARAMETER OF WEIBULL DISTRIBUTION

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ABSTRACT

In the present article, some shrinkage testimators for the scale parameter of a two – parameter Weibull life testing model have been suggested under the LINEX loss function assuming the shape parameter is to be known. The comparisons of the proposed testimators have been made with the improved estimator.

Keywords: Scale Parameter, Weibull distribution, Shrinkage estimator and factor, MSE, Asymmetric loss function, Level of significance.

Notations

β, α	Weibull scale and shape parameter
β_0	Hypothetical value of β
$\hat{\beta}_u$	Unbiased estimate of β
$\hat{\beta}$	MLE estimate of β
a	Shape parameter of the LINEX loss function
MLE	Maximum likelihood estimate
MSE	Mean square error
Δ^*	$\left(\frac{\hat{\beta}_u}{\beta} - 1\right)$
γ_i	$\Gamma\left(n + \frac{i}{\alpha}\right) \ \forall\, i = 0, 1.$
$I(u_1, u_2, v)$	$\int_{u_1}^{u_2} (v) . \frac{e^{-w} w^{n-1}}{\gamma_0} dw$; v may be a function of w
δ	$\frac{\beta_0}{\beta}$
f_0	$a\, c_1 \frac{\gamma_0}{\gamma_1} w^{\frac{1}{\alpha}}$
f_i	$k_i \left(\frac{\gamma_0}{\gamma_1} w^{\frac{1}{\alpha}} - \delta\right)$; $\forall\, i = 1, 2, 3, 4.$
w_i	$\frac{l_i\, \delta^{\alpha}}{2}; \forall\, i = 1, 2.$

1 INTRODUCTION

The Weibull distribution is used in a great variety of applications such as models for life (Weibull, 1951), survival analysis (Berrettoni, 1964), strength, and other properties of many products and materials. Mittnik and Reachev (1993) found that the two – parameter Weibull distribution might be an adequate statistical model for stock returns. In addition, it has been used as a model for diverse items such as ball bearings (Lieblein & Zelen, 1956), vacuum tubes (Kao, 1959), and electrical isolation (Nelson, 1972).

The probability density function of the two-parameter Weibull distribution is given by

$$f(x;\beta,\alpha) = \frac{\alpha}{\beta^{\alpha}} x^{(\alpha-1)} \exp\left[-\left(\frac{x}{\beta}\right)^{\alpha}\right] \quad ; x > 0,\ \beta > 0,\ \alpha > 0. \tag{1.1}$$

Let $x_1,\ x_2,\ \ldots,\ x_n$ be the life times of n items put to test under the Weibull failure model (1.1). Then

$$\hat{\beta} = \left[\frac{1}{n}\sum_{i=1}^{n} x_i^{\alpha}\right]^{\frac{1}{\alpha}} \text{ and } \hat{\beta}_u = n^{\frac{1}{\alpha}} \frac{\gamma_0}{\gamma_1}\hat{\beta}. \tag{1.2}$$

The estimator $\hat{\beta}$ follows a Gamma distribution with the probability density function

$$f(\hat{\beta}) = \frac{\alpha}{\gamma_0}\left[\frac{n}{\beta^{\alpha}}\right]^{n} \hat{\beta}^{(n\alpha-1)} \exp\left[-n\left(\frac{\hat{\beta}}{\beta}\right)^{\alpha}\right] \quad ; \hat{\beta} \geq 0. \tag{1.3}$$

For the special case $\alpha = 1$, the Weibull distribution is the exponential distribution. For $\alpha = 2$, it is the Rayleigh distribution. For shape parameter values in the range $3 \leq \alpha \leq 4$, the shape of the Weibull distribution is close to that of normal distribution, and for a large values of α, say $\alpha \geq 10$, the shape of the Weibull distribution is close to that of the smallest extreme value distribution.

Thompson (1968) suggested a shrinkage estimator $k(\hat{\theta} - \theta_0) + \theta_0$ for any parameter θ and showed that it is more efficient than any usual estimator $\hat{\theta}$ when θ is in the vicinity of θ_0, a guess value of θ. The shrinkage factor $k \in [0,1]$ is specified by the experimenter according to his belief in θ_0. The shrinkage procedure has been applied in numerous problems, including mean survival time in epidemiological studies (Harries & Shakarki, 1979), forecasting of the money supply (Tso, 1990), estimating mortality rates (Marshall, 1991), and improved estimation in sample surveys (Wooff, 1985).

Following Basu and Ebrahimi (1991), the invariant form of the LINEX loss function for $\hat{\beta}_u$ is defined as

$$L(\Delta^*) = e^{a\Delta^*} - a\Delta^* - 1 \ ; \ a \neq 0. \tag{1.4}$$

The shape of this loss function is determined by the value of $'a'$ (the sign of $'a'$ reflects the direction of asymmetry, $a > 0\ (a < 0)$ if overestimation is more (less) serious than the underestimation) and its magnitude reflects the degree of asymmetry.

Pandey et al. (1989) have considered some shrinkage testimator for the shape parameter of the Weibull distribution under the squared error loss function. Singh and Shukla (2000), Montanari et al. (1997), and Hisada and Arizino (2002) have considered the Weibull distribution in different contexts. Pandey and Upadhyay (1985), Nigm (1989), and Dellaportas and Wright (1991) have considered predication problems in two-parameter Weibull distribution. Recently, Prakash and Singh (2008 b) have studied the properties of the Bayes' estimator of the lifetime parameters for two-parameter Weibull distribution. Zellner (1986), Singh et al. (2002), Ahmadi et al.

(2005), Prakash and Singh (2006, 2008 a), Singh et al. (2007), and others have used the LINEX loss function in various estimation and prediction problems.

This paper deals with the some shrinkage testimators for the scale parameter of the two – parameter Weibull distribution when a prior guess value of the scale parameter is available. Assuming the shape parameter is to be known, the relative efficiencies of the proposed testimators are studied with respect to improved estimator of $\hat{\beta}_u$.

2 A CLASS OF ESTIMATORS AND THEIR PROPERTIES

The proposed class of estimators for the unbiased estimator of the parameter β is given by

$$P = c\,\hat{\beta}_u \text{ , where } c \text{ is a constant.} \tag{2.1}$$

The invariant form of the LINEX loss for the class P is

$$L(P) = \exp\left[a\left(\frac{c\,\hat{\beta}_u}{\beta} - 1\right)\right] - a\left(\frac{c\,\hat{\beta}_u}{\beta} - 1\right) - 1$$

and the risk under the invariant form of the LINEX loss is

$$R(P) = e^{-a}\, I\left(0, \infty, \left(\exp\left(a\,c\,\frac{\gamma_0}{\gamma_1}\, w^{\frac{1}{\alpha}}\right)\right)\right) + (a - 1 - a\,c)\ . \tag{2.2}$$

The value of c = c_1 (say), which minimizes the $R(P)$, can be obtained by solving the equation

$$I\left(0, \infty, \left(\exp\left(a\,c\,\frac{\gamma_0}{\gamma_1}\, w^{\frac{1}{\alpha}}\right) w^{\frac{1}{\alpha}}\right)\right) = \gamma_1\, e^{a} \tag{2.3}$$

for a given set of values for n, α and $'a'$ as considered in later calculation.

The minimum risk estimator among the class P is $P_1 = c_1\,\hat{\beta}_u$ with the minimum risk under the invariant form of the LINEX loss

$$R\left(P_1\right) = e^{-a}\, I\left(0, \infty,\ e^{a f_0}\right) - \left(a - 1 - a\,c_1\right). \tag{2.4}$$

Following Thompson (1968), the shrinkage estimator for $\hat{\beta}_u$ is given by

$$Y = k\,(\hat{\beta}_u - \beta_0) + \beta_0\ . \tag{2.5}$$

The value of the shrinkage factor k = k_1 (say), which minimizes the risk of Y under the invariant form of the LINEX loss, may be obtained by solving the equation

$$I\left(0, \infty,\ \frac{f'}{k}\exp\left(a\,f'\right)\right) = (1-\delta)\, e^{a(1-\delta)}\ \ ; f' = k\left(\frac{\gamma_0}{\gamma_1}\, w^{\frac{1}{\alpha}} - \delta\right). \tag{2.6}$$

for a given set of values for $n, \alpha, 'a'$ and δ as considered in later calculation.

The shrinkage estimator Y_1 having minimum risk in the class Y is

$$Y_1 = k_1 (\hat{\beta}_u - \beta_0) + \beta_0 \tag{2.7}$$

with the minimum risk under the invariant form of LINEX loss

$$R(Y_1) = e^{-a(\delta-1)} I(0, \infty, e^{a f_1}) + a(k_1 - 1)(\delta - 1) - 1. \tag{2.8}$$

3 CONCLUSION

The relative bias for the improved shrinkage estimator Y_1 is obtained as

$$RB(Y_1) = \frac{1}{\beta}(E(Y_1) - \beta) = (1 - k_1)(\delta - 1). \tag{3.1}$$

This expression clearly shows that the relative bias is zero at $\delta = 1$ and has a tendency of being negative for $0 < \delta < 1$ and positive for $\delta > 1$.

The relative efficiency for the shrinkage estimator Y_1 with respect to the minimum class of estimators P_1 under the invariant form of the LINEX loss is defined as

$$RE(Y_1, P_1) = \frac{R(P_1)}{R(Y_1)}. \tag{3.2}$$

The expression of $RE(Y_1, P_1)$ is a function of δ, a, n and α. For the selected set of values of $n = 04, 08, 12, 15$; $a = 0.25, 0.50, 1.00, 1.50$; $\delta = 0.40(0.20)1.80$ and $\alpha = 2$, the values of $RE(Y_1, P_1)$ have been calculated (not presented here), and it is observed that the shrinkage estimator Y_1 is more efficient than the improved estimator P_1 when β_0 is in the vicinity of β. More specifically, the shrinkage estimator Y_1 is more efficient than P_1 when $0.40 \le \delta \le 1.60$ and attains maximum efficiency at the point $\delta = 1.00$. The effective interval decreases as n increases and for fixed n, as 'a' increases, the relative efficiency first increases for $\delta < 1.00$ and then decreases.

4 THE SHRINKAGE TESTIMATORS AND THEIR PROPERTIES

We have seen that the shrinkage estimator Y_1 has smaller risk than the estimator P_1 when a hypothetical value of the parameter is in the vicinity of the true value. This suggests that when β_0 for β is given, the hypothesis $H_0 : \beta = \beta_0$ against $H_1 : \beta \ne \beta_0$ is carried out first and upon the acceptance of the H_0, the shrinkage estimator Y_1 is used as an estimator for β; otherwise P_1 as an estimator for β. Thus the proposed shrinkage testimator for β is given by

$$T_1 = \begin{cases} k_1(\hat{\beta}_u - \beta_0) + \beta_0 & \text{if } t_1 \le \hat{\beta}_u \le t_2 \\ c_1 \hat{\beta}_u & \text{otherwise} \end{cases}, \tag{4.1}$$

where $t_1 = \frac{\gamma_0}{\gamma_1}\left(\frac{\beta_0^\alpha l_1}{2}\right)^{\frac{1}{\alpha}}$, $t_2 = \frac{\gamma_0}{\gamma_1}\left(\frac{\beta_0^\alpha l_2}{2}\right)^{\frac{1}{\alpha}}$ and l_1, l_2 being the values of the lower and upper

$100\left(\varepsilon/2\right)\%$ points of the chi – square distribution with $2n$ degrees of freedom at ε level of significance.

The expressions of the relative bias and risk under the invariant form of the LINEX loss for the proposed shrinkage testimator are obtained as

$$RB\left(T_1\right) = I\left(w_1, w_2, \left(f_1 - f_0 + \delta\right)\right) + c_1 - 1 \tag{4.2}$$

and

$$R\left(T_1\right) = e^{a(\delta-1)} I\left(w_1, w_2, e^{af_1}\right) + e^{-a} I\left(0, \infty, e^{af_0}\right) - e^{-a} I\left(w_1, w_2, e^{af_0}\right)$$
$$+ a I\left(w_1, w_2, \left(f_0 - f_1 - \delta\right)\right) + a\left(1 - c_1\right) - 1. \tag{4.3}$$

Waikar et al. (1984) have suggested the idea of taking shrinkage factor as a function of the test statistic. Under $H_0 : \beta = \beta_0$

$$l_1 \le 2n\left(\frac{\hat{\beta}}{\beta_0}\right)^{\alpha} \le l_2 \Leftrightarrow 0 \le \frac{1}{l_2 - l_1}\left(2n\left(\frac{\hat{\beta}}{\beta_0}\right)^{\alpha} - l_1\right) = k_2\,(\text{say}) \le 1\,. \tag{4.4}$$

Based upon this shrinkage factor k_2, the shrinkage testimator is given by

$$T_2 = \begin{cases} k_2(\hat{\beta}_u - \beta_0) + \beta_0 & \text{if } t_1 \le \hat{\beta}_u \le t_2 \\ c_1 \hat{\beta}_u & \text{otherwise} \end{cases}. \tag{4.5}$$

When $H_0 : \beta = \beta_0$ is accepted, $l_1 \le 2n \le l_2 \Rightarrow \frac{l_1}{2n} \le 1$. If there is interest in smaller values of the shrinkage factor k, then one can use $\frac{l_1}{2n} \cong 1$. Thus the shrinkage testimator is given by

$$T_3 = \begin{cases} k_3(\hat{\beta}_u - \beta_0) + \beta_0 & \text{if } t_1 \le \hat{\beta}_u \le t_2 \\ c_1 \hat{\beta}_u & \text{otherwise} \end{cases}. \tag{4.6}$$

Here $k_3 = \frac{2n}{l_2 - l_1}\left|\left(\hat{\beta}/\beta_0\right)^{\alpha} - 1\right|$, it may possible that the value of the shrinkage factor is negative, so we make it positive. Adke et al. (1987) and Pandey et al. (1988) have considered this type of shrinkage factor.

As the value of c_1 also lies between zero and one, it may be a choice for the shrinkage factor. Based on this, the shrinkage testimator is defined as

$$T_4 = \begin{cases} c_1(\hat{\beta}_u - \beta_0) + \beta_0 & \text{if } t_1 \le \hat{\beta}_u \le t_2 \\ c_1 \hat{\beta}_u & \text{otherwise} \end{cases}. \tag{4.7}$$

The expressions of the relative biases and risk under the invariant form of the LINEX loss function for these shrinkage testimators are given as

$$RB\left(T_i\right) = I\left(w_1, w_2, \left(f_i - f_0 + \delta\right)\right) + c_1 - 1 \tag{4.8}$$

and

$$R\left(T_i\right) = e^{a(\delta-1)} I\left(w_1, w_2, e^{af_i}\right) + e^{-a} I\left(0, \infty, e^{af_0}\right) - e^{-a} I\left(w_1, w_2, e^{af_0}\right)$$

$$+ a I(w_1, w_2, (f_0 - f_i - \delta)) + a(1 - c_1) - 1; \quad (4.9)$$

where $k_4 = c_1$ (say) and $i = 2, 3, 4.$

5 CONCLUSION AND RECOMMENDATIONS

The relative efficiencies of $T_i\ ; i = 1, 2, ..., 4,$ with respect to the minimum risk estimator P_1 are given by,

$$RE(T_i, P_1) = \frac{R(P_1)}{R(T_i)} \ ; i = 1, ..., 4.$$

The expressions of the relative biases and the $RE(T_i, P_1); i = 1, ..., 4$ are the function of $\delta,\ a,\ n,\ \alpha$ and ε. The Tables 1 – 4 show the values of $RE(T_i, P_1); i = 1, ..., 4$ for the same set of values of $\delta,\ a,\ n$ and α as considered earlier with $\varepsilon = 0.01$ and 0.05. The numerical findings are presented here only for the relative efficiency.

The relative biases are negligibly small and lie between -0.043 to 0.056 for the testimator T_1 and -0.039 to 0.02 for testimator T_2. The absolute values of biases decrease as the sample size n increases. Further, $|RB(T_2)|$ increases when level of significance ε increases in $0.50 \le \delta \le 1.00$ and decreases otherwise. A similar trend has been seen for T_1 when $0.50 \le \delta \le 0.90$. The relative bias of T_3 lies between -0.038 to 0.027 and for T_4 in -0.045 to 0.212 and are negligible small. The absolute values of biases decrease as the sample size n increases. In addition, $|RB(T_3)|$ increases as ε increases in $0.50 \le \delta \le 0.90$ and decreases otherwise. On the other hand, $|RB(T_4)|$ decreases as ε increases except for $\delta = 1$.

The shrinkage testimators T_1 and T_4 perform better for all considered values of the parametric space. On the other hand, the shrinkage testimators T_2 and T_3 are efficient when $0.40 \le \delta \le 1.40$. All the shrinkage testimators attain maximum efficiency at the point $\delta = 1.00$. For fixed ε and 'a', as the sample size increases, the relative efficiency decreases in $0.40 \le \delta \le 1.60$ for the testimators T_1 and T_3 whereas it decreases for T_2 in the entire range of δ. For the shrinkage testimator T_4, the relative efficiency decreases as n increases when $\delta < 1$.

Table 1. Relative efficiency of testimators $T_1 - T_4$ for n=4 items for a variety of ε, δ, and 'a' parameters

ε = 0.01		δ							
a	n = 04	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80
0.25	T_1	1.0388	1.4231	2.9039	23.129	3.2006	1.7112	1.4369	1.3442
	T_2	1.0436	2.8075	8.7184	15.045	5.5219	2.8115	0.7667	0.3997
	T_3	1.0291	2.2053	6.2012	31.971	6.8515	2.7527	1.3971	0.8760
	T_4	1.0326	1.8530	2.4589	2.7623	2.0545	1.8572	1.6698	1.4687
0.50	T_1	1.0384	1.4209	2.9474	24.472	3.1285	1.6562	1.3925	1.3045
	T_2	1.0420	2.7816	8.6197	14.689	5.3343	2.7392	0.7257	0.3720
	T_3	1.0283	2.1939	6.1556	32.208	6.6751	2.6465	1.3287	0.8253
	T_4	1.0321	1.8600	2.5152	2.8492	2.0502	1.8215	1.6125	1.3974
1.00	T_1	1.0348	1.4011	2.8664	22.557	2.8514	1.4842	1.2511	1.1816
	T_2	1.0391	2.7328	8.3030	14.271	4.7606	2.5356	0.6222	0.3075
	T_3	1.0262	2.1619	5.9448	30.516	6.0623	2.3439	1.1501	0.6998
	T_4	1.0297	1.8376	2.5364	2.8139	1.9111	1.6631	1.4514	1.2436
1.50	T_1	1.0326	1.3930	2.9160	24.556	2.7010	1.3731	1.2267	1.0961
	T_2	1.0365	2.6884	8.0946	14.490	4.3963	2.4018	0.5505	0.2615
	T_3	1.0248	2.1404	5.8359	30.504	5.7097	2.1469	1.0278	0.6114
	T_4	1.0286	1.8415	2.5548	2.9433	1.8629	1.5648	1.3269	1.1077
ε = 0.05									
0.25	T_1	1.0158	1.2276	2.5369	20.555	3.4577	1.7466	1.4176	1.3063
	T_2	1.0175	2.2503	5.3586	26.547	7.3278	3.0179	0.8259	0.4218
	T_3	1.0144	1.9357	4.2464	21.452	7.5991	3.2837	1.8379	1.2641
	T_4	1.0138	1.6206	2.5293	3.0479	2.3386	1.8986	1.6464	1.4191
0.50	T_1	1.0153	1.2239	2.5323	22.007	3.4097	1.6973	1.3770	1.2743
	T_2	1.0167	2.2348	5.2537	25.852	7.1283	2.9416	0.7835	0.3935
	T_3	1.0138	1.9248	4.1779	21.276	7.4640	3.1655	1.7527	1.1965
	T_4	1.0135	1.6208	2.5744	3.1663	2.3622	1.8683	1.5921	1.3534
1.00	T_1	1.0140	1.2115	2.4577	20.442	3.1194	1.5302	1.2426	1.1589
	T_2	1.0154	2.2093	5.0861	24.043	6.4299	2.7213	0.6747	0.3268
	T_3	1.0127	1.9047	4.0512	20.473	6.8594	2.8158	1.5249	1.0242
	T_4	1.0124	1.6063	2.5336	3.1032	2.2343	1.7154	1.4379	1.2109
1.50	T_1	1.0131	1.2039	2.4346	22.456	2.9987	1.4269	1.2038	1.0837
	T_2	1.0142	2.1842	4.9117	22.679	6.0204	2.5782	0.5995	0.2791
	T_3	1.0118	1.8868	3.9330	19.971	6.5513	2.5907	1.3690	0.9027
	T_4	1.0118	1.6033	2.5885	3.2823	2.2320	1.6261	1.3182	1.0828

Table 2. Relative efficiency of testimators $T_1 - T_4$ for n=8 items for a variety of ε, δ, and 'a' parameters

ε = 0.01		δ							
a	n = 08	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80
0.25	T_1	1.0007	1.0692	1.7534	10.113	2.1249	1.3952	1.2663	1.2185
	T_2	1.0008	1.9249	4.0749	12.988	3.7909	1.9612	0.3600	0.1761
	T_3	1.0005	1.6643	3.1006	19.502	3.3278	1.4490	0.8452	0.6015
	T_4	1.0006	1.4543	2.3680	2.8256	2.1000	1.8675	1.6830	1.4765
0.50	T_1	1.0007	1.0720	1.8360	12.757	2.1973	1.4311	1.3024	1.2564
	T_2	1.0008	1.9227	4.1165	14.281	3.9223	1.9816	0.3619	0.1739
	T_3	1.0005	1.6664	3.1555	22.489	3.4523	1.4791	0.8555	0.6052
	T_4	1.0006	1.4575	2.4863	3.1678	2.2404	1.9329	1.6938	1.4440
1.00	T_1	1.0006	1.0705	1.8655	14.575	2.1519	1.3901	1.2248	1.2345
	T_2	1.0007	1.9155	4.0733	13.636	3.8111	1.9344	0.3335	0.1542
	T_3	1.0004	1.6638	3.1416	24.092	3.3737	1.4065	0.7993	0.5582
	T_4	1.0006	1.4553	2.4252	3.3891	2.2629	1.8860	1.6078	1.3369
1.50	T_1	1.0006	1.0685	1.8728	15.827	2.0764	1.3314	1.1759	1.1969
	T_2	1.0006	1.9085	4.0144	12.610	3.6456	1.8761	0.3020	0.1340
	T_3	1.0004	1.6605	3.1105	24.944	3.2471	1.3172	0.7345	0.5056
	T_4	1.0006	1.4522	2.4913	3.5327	2.2368	1.8064	1.5048	1.2266
ε = 0.05									
0.25	T_1	1.0002	1.0255	1.4727	9.5107	2.4519	1.4155	1.2310	1.1313
	T_2	1.0002	1.8216	2.7596	14.814	5.6484	2.0723	0.3755	0.1799
	T_3	1.0002	1.6198	2.4840	11.723	3.7168	1.7997	1.2264	1.0192
	T_4	1.0002	1.3915	1.8741	3.2563	2.4978	1.8999	1.6259	1.3381
0.50	T_1	1.0002	1.0260	1.4899	11.516	2.5966	1.4585	1.2673	1.1684
	T_2	1.0002	1.8201	2.7458	15.388	5.9607	2.0975	0.3779	0.1778
	T_3	1.0002	1.6184	2.4693	12.264	3.9142	1.8446	1.2467	1.0325
	T_4	1.0002	1.3922	1.9091	3.6549	2.7329	1.9731	1.6357	1.3097
1.00	T_1	1.0001	1.0251	1.4850	12.471	2.6003	1.4255	1.2092	1.1577
	T_2	1.0002	1.8168	2.7067	15.166	5.9018	2.0489	0.3491	0.1579
	T_3	1.0002	1.6156	2.4320	12.249	3.8859	1.7661	1.1754	0.9676
	T_4	1.0001	1.3910	1.9136	3.9108	2.8332	1.9350	1.5578	1.2234
1.50	T_1	1.0001	1.0239	1.4746	12.847	2.5538	1.3737	1.1566	1.1325
	T_2	1.0001	1.8139	2.6689	14.759	5.7354	1.9874	0.3169	0.1373
	T_3	1.0001	1.6129	2.3965	12.064	3.7892	1.6642	1.0894	0.8904
	T_4	1.0001	1.3895	1.9080	4.0734	2.8653	1.8627	1.4632	1.1330

Table 3. Relative efficiency of testimators $T_1 – T_4$ for n=12 items for a variety of ε, δ, and 'a' parameters

$\varepsilon = 0.01$		δ							
a	$n = 12$	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80
0.25	T_1	1.0000	1.0107	1.3810	9.0131	1.8983	1.3356	1.2558	1.2103
	T_2	1.0000	1.7897	2.6631	11.839	3.3273	1.6866	0.2362	0.1110
	T_3	1.0000	1.5872	2.2885	14.802	2.3208	1.1252	0.7520	0.6090
	T_4	1.0000	1.3737	1.9624	4.1537	2.6504	2.1825	1.8657	1.5273
0.50	T_1	1.0000	1.0103	1.3698	8.7289	1.8437	1.3008	1.2232	1.1835
	T_2	1.0000	1.7890	2.6460	11.582	3.2192	1.6577	0.2226	0.1027
	T_3	1.0000	1.5867	2.2739	14.490	2.2457	1.0776	0.7156	0.5776
	T_4	1.0000	1.3732	1.9514	4.1134	2.5862	2.1133	1.7999	1.4713
1.00	T_1	1.0000	1.0100	1.3804	9.7685	1.8232	1.2863	1.2101	1.1765
	T_2	1.0000	1.7880	2.6337	12.161	3.1721	1.6320	0.2066	0.0916
	T_3	1.0000	1.5862	2.2665	15.531	2.2113	1.0347	0.6781	0.5432
	T_4	1.0000	1.3724	1.9489	4.3748	2.6178	2.0677	1.7192	1.3779
1.50	T_1	1.0000	1.0096	1.3767	10.024	1.7617	1.2382	1.1525	1.1459
	T_2	1.0000	1.7869	2.6113	12.185	3.0462	1.5930	0.1868	0.0793
	T_3	1.0000	1.5855	2.2489	15.729	2.1231	1.0696	0.6264	0.4976
	T_4	1.0000	1.3716	1.9366	4.4644	2.5641	1.9733	1.6148	1.2793
$\varepsilon = 0.05$									
0.25	T_1	1.0000	1.0026	1.1794	6.9721	2.3640	1.3564	1.2046	1.0721
	T_2	1.0000	1.7727	2.1018	8.0810	5.4666	1.7601	0.2404	0.1133
	T_3	1.0000	1.5776	1.9338	7.6355	2.7304	1.4393	1.1536	1.1180
	T_4	1.0000	1.3612	1.5852	3.6812	3.4603	2.2225	1.7488	1.2633
0.50	T_1	1.0000	1.0025	1.1679	6.7855	2.3031	1.3229	1.1785	1.0475
	T_2	1.0000	1.7725	2.0944	7.9461	5.3071	1.7291	0.2267	0.1048
	T_3	1.0000	1.5774	1.9262	7.5270	2.6554	1.3853	1.1074	1.0766
	T_4	1.0000	1.3611	1.5803	3.6472	3.3954	2.1557	1.6932	1.2274
1.00	T_1	1.0000	1.0024	1.1713	7.0083	2.3346	1.3128	1.1698	1.0535
	T_2	1.0000	1.7722	2.0837	7.9268	5.3496	1.7028	0.2106	0.0935
	T_3	1.0000	1.5770	1.9120	7.5359	2.6608	1.3436	1.0660	1.0391
	T_4	1.0000	1.3609	1.5756	3.7547	3.5276	2.1180	1.6250	1.1652
1.50	T_1	1.0000	1.0023	1.1679	6.9191	2.2942	1.2697	1.1383	1.0356
	T_2	1.0000	1.7718	2.0720	7.7809	5.2146	1.6613	0.1906	0.0809
	T_3	1.0000	1.5766	1.8986	7.4238	2.5882	1.2707	1.0001	0.9779
	T_4	1.0000	1.3606	1.5687	3.7680	3.5221	2.0293	1.5345	1.0974

Table 4. Relative efficiency of testimators $T_1 - T_4$ for n=12 items for a variety of ε, δ, and 'a' parameters

$\varepsilon = 0.01$		δ							
a	$n = 15$	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80
0.25	T_1	1.0000	1.0020	1.1883	6.2381	1.6409	1.2275	1.1616	1.1151
	T_2	1.0000	1.7717	2.1949	8.1917	2.8204	1.5138	0.1679	0.0773
	T_3	1.0000	1.5753	1.9690	9.7082	1.7304	1.0869	0.6777	0.6081
	T_4	1.0000	1.3608	1.6955	4.5675	2.9160	2.2982	1.9261	1.5201
0.50	T_1	1.0000	1.0020	1.1876	6.0691	1.6258	1.2021	1.1384	1.0973
	T_2	1.0000	1.7715	2.1862	8.0437	2.7436	1.4946	0.1589	0.0718
	T_3	1.0000	1.5751	1.9607	9.5171	1.6817	1.0728	0.6493	0.5816
	T_4	1.0000	1.3607	1.6893	4.5292	2.8515	2.2299	1.8629	1.4697
1.00	T_1	1.0000	1.0019	1.1920	6.5635	1.6106	1.1827	1.1258	1.0902
	T_2	1.0000	1.7713	2.1830	8.4112	2.7063	1.4747	0.1471	0.0638
	T_3	1.0000	1.5750	1.9564	10.070	1.6526	1.0276	0.6169	0.5502
	T_4	1.0000	1.3605	1.6825	4.7151	2.8715	2.1746	1.7800	1.3840
1.50	T_1	1.0000	1.0018	1.1858	6.3532	1.5427	1.1357	1.0863	1.0579
	T_2	1.0000	1.7711	2.1691	8.2266	2.5791	1.4420	0.1319	0.0546
	T_3	1.0000	1.5748	1.9430	9.8388	1.5711	1.0339	0.5679	0.5041
	T_4	1.0000	1.3603	1.6720	4.6800	2.7658	2.0549	1.6671	1.2916
$\varepsilon = 0.05$									
0.25	T_1	1.0000	1.0004	1.0756	4.6211	2.0576	1.2421	1.1085	1.0580
	T_2	1.0000	1.7681	1.9133	5.2811	4.7292	1.5653	0.1689	0.0803
	T_3	1.0000	1.5728	1.7502	5.3810	2.1051	1.1785	1.0241	1.0636
	T_4	1.0000	1.3579	1.4674	4.0146	3.2751	2.3401	1.7663	1.1830
0.50	T_1	1.0000	1.0004	1.0773	4.5204	2.0500	1.2176	1.0904	1.0543
	T_2	1.0000	1.7681	1.9098	5.2087	4.6130	1.5448	0.1599	0.0745
	T_3	1.0000	1.5727	1.7462	5.3122	2.0575	1.1424	0.9940	1.0390
	T_4	1.0000	1.3579	1.4650	3.9480	3.2481	2.2743	1.7156	1.1545
1.00	T_1	1.0000	1.0004	1.0774	4.5979	2.0811	1.2027	1.0808	1.0537
	T_2	1.0000	1.7680	1.9060	5.2316	4.6618	1.5242	0.1481	0.0662
	T_3	1.0000	1.5727	1.7397	5.3411	2.0612	1.1118	0.9674	1.0197
	T_4	1.0000	1.3579	1.4615	4.0737	3.2708	2.2262	1.6491	1.1046
1.50	T_1	1.0000	1.0003	1.0741	4.4511	2.0099	1.1581	1.0498	1.0372
	T_2	1.0000	1.7680	1.9002	5.1220	4.4827	1.4892	0.1327	0.0566
	T_3	1.0000	1.5726	1.7328	5.2362	1.9832	1.0480	0.9119	0.9723
	T_4	1.0000	1.3578	1.4574	3.9790	3.2307	2.1107	1.5554	1.0483

6 REFERENCES

Adke, S. R., Waikar ,V. B., & Schuurmann, F. J. (1987) A two stage shrinkage testimator for the mean of an Exponential distribution. *Communication in Statistics – Theory and Methods,* 16, 1821 – 1834.

Ahmadi, J., Doostparast, M., & Parsian, A. (2005) Estimation and prediction in a two – parameter Exponential distribution based on k - record values under LINEX loss function. *Communications in Statistics –Theory and Methods, 34,* 795 – 805.

Basu, A. P. & Ebrahimi, N. (1991) Bayesian approach to life testing and reliability estimation using asymmetric loss function. Journal of Statistical Planning and Inferences, 29, 21 – 31.

Berrettoni, J. A. (1964) Practical applications of the Weibull distribution. *Industrial Quality Control, 21*, 71 – 79.

Dallaportas, P. & Wright, D. E. (1991) Numerical prediction for the two-parameter Weibull distribution. *The Statistician, 40*, 365 – 372.

Harris, E. & Shakarki, G. (1979) Use of the population distribution to improve estimation of individual mean in epidemiological studies. *Journal of Chronic Disease, 32*, 233 – 243.

Hisada, K. & Arizino, I. (2002) Reliability Tests for Weibull Distribution With Varying Shape-Parameter Based on Complete Data. *IEEE Transactions on Reliability, 51 (3*), 331 – 336.

Kao, J. H. K. (1959) A graphical estimation of mixed Weibull parameters in life – testing electron tubes. *Technometrics, 4*, 309 – 407.

Lieblein, J. & Zelen, M. (1956) Statistical investigation of the fatigue life of deep groove ball bearings. *Journal of Res. Nat. Bur. Std., 57,* 273 – 315.

Marshall, R. J. (1991) Mapping disease and mortality rates using empirical Bayes estimators. *Journal of Applied Statistics, 40,* 283 – 294.

Mittnik, S. & Rachev, S. T. (1993) Modeling asset returns with alternative stable distribution. *Economic Reviews, 12,* 261 – 330.

Montanari, G. C., Mazzanti, G., Cacciari, J., & Fothergill, J.C. (1997) In search of convenient techniques for reducing bias in the estimation of Weibull parameters for uncensored tests. *IEEE Transactions on Dielectrics and Electrical Insulation, 4(3),* 306 – 313.

Nelson, W. B. (1972) Graphical analysis of accelerated life test data with the inverse power law model. *IEEE Transaction on Reliability, R 21*, 2 – 11.

Nigm, A. M. (1989) An informative Bayesian prediction for the Weibull lifetime distribution. *Communication Statistics – Theory and Methods, 18,* 897 – 911.

Pandey, B. N., Malik, H. J., & Srivastava, R. (1988) Shrinkage testimator for the variance of a normal distribution at single and double stages. *Microelectron Reliability, 28 (6),* 929 – 944.

Pandey, B. N., Malik, H. J., & Srivastava, R. (1989). Shrinkage testimators for the shape parameter of Weibull distribution under type II censoring. *Communications in Statistics -Theory and Methods, 18*, 1175-1199.

Pandey, M. & Upadhyay, S. K. (1985) Bayes shrinkage estimators of Weibull parameters. *IEEE Transactions on Reliability, 34*, 491 – 494.

Prakash, G. & Singh, D. C. (2006) Shrinkage testimators for the inverse dispersion of the inverse Gaussian distribution under the LINEX loss function. *Austrian Journal of Statistics, 35 (4)*, 463 – 470.

Prakash, G. & Singh, D. C. (2008 a) Shrinkage estimation in Exponential type – II censored data under the LINEX loss function. *Journal of the Korean Statistical Society, 37 (1)*, 53 - 61.

Prakash, G. & Singh, D. C. (2008 b) Item Failure Data of Weibull Failure Model Under Bayesian Estimation. *Journal of Statistical Research* (To appear).

Singh, D. C., Prakash, G. & Singh, P. (2007) Shrinkage testimators for the shape parameter of Pareto distribution using the LINEX loss function. *Communication in Statistics Theory and Methods, 36 (4)*, 741-753.

Singh, H. P. & Shukla, S. K. (2000) Estimation in the two-parameter Weibull distribution with prior information. *IAPQR Transactions, 25 (2)*, 107 - 118.

Singh, U., Gupta, P. K., & Upadhyay, S. K. (2002). Estimation of exponentiated Weibull shape parameters under LINEX loss function. *Communication in statistics – Simulation, 31(4)*, 523 – 537.

Thompson, J. R. (1968) Some shrinkage techniques for estimating the mean. *Journal of the American Statistical Association, 63*, 113 - 122.

Tso, G. (1990) Forecasting money supply in Hong Kong with a multiple shrinkage estimator. *Proceeding of the ASA Section on Business and Commerce.*

Waikar, V. B., Schuurmann, F. J., & Raghunathan,T. E. (1984) On a two stage shrinkage testimator of the mean of a normal distribution. *Communications in Statistics – Theory and Methods, 13*, 1901 - 1913.

Weibull, W. (1951) A statistical distribution functions of wide applicability. *Journal of Applied Mathematics and Mechanics, 18*, 293 – 297.

Wooff, D. A. (1985) Bounds on reciprocal moments with applications and developments in Stein estimation and post stratification. *Journal of Royal Statistical Society, B – 47*, 362 – 371.

Zellner, A. (1986). Bayesian estimation and prediction using asymmetric loss function. *Journal of the American Statistical Association, 81*, 446-451.