BAYES ESTIMATORS OF EXPONENTIAL PARAMETERS FROM A CENSORED SAMPLE USING A GUESSED ESTIMATE

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ABSTRACT

This paper provides the Bayes estimators of the failure rate and reliability function for a one-parameter, exponential distribution by utilizing a point guess estimate of the parameter. For deriving the Bayes estimators, the prior distributions are chosen such that they are centered at the known prior values of parameters. The validity of proposed estimators is examined with respect to their maximum likelihood estimators (MLE) and Thompson's Shrinkage estimator on the basis of Monte Carlo simulations of 1000 samples.

1 INTRODUCTION

In life testing experiments, experimenters often possess certain information about a parameter of interest, through past experience or familiarity with the experiment. The most common type of information is a probable value of the parameter θ, say θ_0. This θ_0 has been referred in statistical literature as the *point guess* about θ. The use of the point guess for inferences regarding a parameter has been considered by many authors. Perhaps the most popular technique that utilizes the knowledge of point guess is the *shrinkage technique*, originally suggested by Thompson (1968). He suggested shrinking the usual estimator $\hat{\theta}$ of θ towards the prior guess value θ_0 and proposed the shrunken estimator T = $k\hat{\theta}+(1-k)\theta_0$, where k is a constant $(0 \le k \le 1)$. (1-k) represents the experimenter's belief in the guess value θ_0. It was found that the estimator T is more efficient than $\hat{\theta}$ if the true value of θ is close to θ_0 but may be less efficient otherwise. The technique was further modified by Mehata and Srinivasan (1971), Pandey (1979), and Lemmer (1981). Point guess in Bayesian point estimation problems has been used by Lemmer (1981) and Pandey and Upadhyay (1985), among others. These authors have postulated a prior distribution for θ, which places a weight (1-k) at θ_0 and distributes the rest of the mass k according to some distribution $g(\theta)$. Such a prior has a point of discontinuity at θ_0. Moreover, a sudden change in the confidence for a value close to θ_0 seems to be unrealistic. Another Bayesian estimation procedure as an alternative to the shrinkage estimation procedure that utilizes the point guess θ_0 has been suggested by Pandey and Srivastava (1985) and Pandey and Upadhyay (1988). Their technique chooses a subfamily from a family of priors such that the mean of the prior distribution is equal to θ_0. Equalization of the mean to θ_0, however, does not seem appropriate unless the prior guess θ_0 is specified as the average value of θ. As mentioned above, prior to the sample selection, the experimenter believes that the point guess θ_0 is a possible value of the parameter. Thus, it seems more appropriate to interpret θ_0 as the most probable value rather than interpreting it as the average value of θ. Therefore, we propose that the choice of subfamily of the prior be made by equalizing the mode of the prior distribution to θ_0. As an illustration of our proposition, we have considered the problem of estimation of the failure rate and reliability for a one-parameter exponential distribution.

The exponential distribution is a widely used model in a variety of statistical procedures. Among its most prominent applications are those in the field of life testing and reliability problems.

The probability density function (pdf) of one parameter exponential distribution is

$$f\left(X,\lambda\right)=\lambda\, e^{-X\lambda}; \qquad X>o, \lambda>o. \tag{1}$$

where λ is the failure rate. The reliability function R(t) which is probability of survival for at least time t is given by

$$R\left(t\right)=e^{-\lambda t}\,. \tag{2}$$

The Bayesian and classical inferences regarding parameter λ and R(t) have been considered by many authors (Martz & Waller, 1982; Sinha, 1985)). In the next section, we obtain the Bayes estimator of λ and R(t) under a quadratic loss function when prior distribution is chosen as mentioned above. In Section 3, performance of the proposed estimators is compared with the corresponding MLE and shrinkage estimators on the basis of a Monte Carlo simulation study.

2 ESTIMATION OF EXPONENTIAL PARAMETERS

We assume that *n* items are tested and the experiment is continued until the first r failures are observed (type II censoring). If $\underline{X}=\left(X_1,X_2,....X_r\right)$; $\left(r\le n\right)$ denotes the first *r* ordered observations, then the likelihood function is given by

$$L\left(\underline{X}/\lambda\right)=\frac{n!}{\left(n-r\right)!}\lambda^r \quad \exp\left(-\lambda S_r\right); \qquad \lambda>o. \tag{3}$$

Where $S_r=\sum_{i=1}^{n} X_i+\left(n\text{-}r\right)X_r$ and is interpreted as 'total observed failure times.'

2.1 Estimation of λ

It is well known that the MLE of λ is

$$\hat{\lambda}_m=\frac{r}{S_r}\,. \tag{4}$$

Suppose that prior to the sample information $\underline{X}$, some *a priori* information including a point guess λ_0 regarding λ is also available. In such a situation, one may be willing to use a shrinkage estimator which is defined below:

$$\hat{\lambda}_S=k\,\hat{\lambda}_m+\left(1\text{-}k\right)\lambda_0, \tag{5}$$

where (1-k) is the confidence in λ_0 as prespecified by experimenter. The estimator $\hat{\lambda}_S$ performs better than $\hat{\lambda}_m$ if λ is close to λ_0 , but in other situations it be may worse than $\hat{\lambda}_m$. The shrinkage estimator utilizes only a single piece of prior information in the form of a point guess, which may always be questionable. Perhaps a better and more easily justifiable way of utilizing the prior information would be by the Bayes method of selecting a family of priors that can describe a variety of information. One such family of priors is the natural conjugate prior (see Raiffa & Schlair, 1961). The natural conjugate prior for λ is

$$g_1(\lambda)=\frac{a^C}{\Gamma c}\lambda^{c-1}\exp(-a\lambda); \qquad c,a>o,\lambda>o. \tag{6}$$

The mode of this distribution is at a/(c-1). In the presence of point guess λ_0 about λ, a subfamily of (6) may be obtained by choosing a and c such that $a/(c-1)=\lambda_0$, i.e., $a=\lambda_0/(c-1)$ which results in

$$g_2(\lambda)=\frac{[(c-1)/\lambda_0]}{\Gamma c}\lambda^{c-1}\exp\{-(c-1)\lambda/\lambda_0\};\ c>1,\lambda>0. \tag{7}$$

The posterior distribution of λ can easily be obtained by combining (3) and (7) by the Bayes rule as

$$P(\lambda/\underline{X})=\frac{[(c-1)/\lambda_0+S_r]^{r+c}}{\Gamma(r+c)}\lambda^{r+c-1}\exp\left[-\lambda\left\{\frac{(c-1)}{\lambda_0}+S_r\right\}\right]. \tag{8}$$

Hence the Bayes estimator of λ, denoted by $\hat{\lambda}_P$ under a quadratic loss function, which is the mean of the posterior distribution function from (8) is

$$\hat{\lambda}_P=\frac{(r+c)\lambda_0}{\lambda_0 S_r+(c-1)}. \tag{9}$$

2.2 Estimation of R(t)

In this subsection, we obtain the Bayes estimator of the reliability function $R(t)=e^{-\lambda t}$, which is the probability that a unit will survive until a specified time *t*. If we make the transformation

$$\lambda=-\frac{1}{t}\log R,$$

where R=R(t), the density and likelihood function, given in (1) and (3) respectively, can be written in terms of R as

$$f(X/R)=R^{x/t}\left(-\frac{1}{t}\log R\right); \qquad 0\le R\le 1 \tag{10}$$

and

$$L(\underline{X}/R)=\frac{n!}{(n-r)!}\left(-\frac{1}{t}\log R\right)^r R^{(S_r/t)}; \quad 0\le R\le 1. \tag{11}$$

A flexible and rich prior (in the sense of being capable of describing a variety of information) is the Beta Prior (see Martz & Waller, 1985; Canfield, 1970). If the point guess regarding the reliability at time *t* is R_0, and equating it to the mode of Beta Prior, we get a subfamily of priors as

$$g(R)\alpha R^{\frac{R_0(\beta_0-1)}{1-R_0}}(1-R)^{\beta_0-1}; \qquad 0\le R\le 1, \beta_0>2. \tag{12}$$

The posterior distribution of R given $\underline{X}$ can be obtain from the following

$$P\left(R/\underline{X}\right)=\frac{L\left(\underline{X}/R\right)g\left(R\right)}{\int_0^1 L\left(\underline{X}/R\right)g\left(R\right)\,dR}. \tag{13}$$

Substituting $L\left(\underline{X}/R\right)$ and g(R) from (11) and (12) in (13) and simplifying under the assumption that β_0 is integer, we have

$$P\left(R/\underline{X}\right)=\frac{\left(-\log R\right)^r R^{\left\{\frac{S_r}{t}+\frac{R_0(\beta_0-1)}{1-R_0}\right\}}\left(1-R_0\right)^{\beta_0-1}}{\Gamma\left(r+1\right)\sum_{j=0}^{\beta_0}\left[\binom{\beta_0-1}{j}\left(-1\right)^j\left\{\frac{S_r}{t}+\frac{R_0\left(\beta_0-1\right)}{1-R_0}+j+1\right\}^{-(r+1)}\right]}\; 0\le R\le 1 \;. \tag{14}$$

Under the quadratic loss function, the Bayes estimator of R, $\hat{R}_P$, that is the mean of (14) is

$$\hat{R}_P=\frac{\sum_{j=0}^{\beta_0}\left[\binom{\beta_0-1}{j}\left(-1\right)^j\left\{\frac{S_r}{t}+\frac{R_0\left(\beta_0-1\right)}{1-R_0}+i+2\right\}^{-(r+1)}\right]}{\sum_{j=0}^{\beta_0}\left[\binom{\beta_0-1}{j}\left(-1\right)^j\left\{\frac{S_r}{t}+\frac{R_0\left(\beta_0-1\right)}{1-R_0}+i+1\right\}^{-(r+1)}\right]}. \tag{15}$$

The MLE of R, $\hat{R}_m$, is

$$\hat{R}_m=\exp\left(-\frac{rt}{S_r}\right). \tag{16}$$

The shrinkage estimator of R, $\hat{R}_S$, with confidence (1-k) in R_0 can be defined as

$$\hat{R}_S=K\exp\left(-\frac{rt}{S_r}\right)+\left(1-k\right)R_o. \tag{17}$$

3 COMPARISON OF ESTIMATORS

In the previous section, Bayes estimators of λ and R(t) have been obtained as alternative estimators to shrinkage estimators, utilizing the knowledge of a point guess. Now the question of comparison of the proposed estimators with the corresponding shrinkage and MLE estimators arises. For comparison, we propose the risk criterion, which is well accepted by non Bayesians. It is well known that the risk of an estimator T_1 of parameter θ is defined as

$$RE\left(\theta,T_1\right)=E\left(T_1-\theta\right)^2, \tag{18}$$

where the expectation is taken over the whole sample space. Naturally, T_1 will be superior to another estimator T_2 of θ if $R(\theta, T_2) > R(\theta, T_1)$, or in other words, we can say the risk efficiency $RE(T_1, T_2) > 1$, where

$$RE\left(T_1, T_2\right)=\frac{RE\left(\theta, T_2\right)}{RE\left(\theta, T_1\right)}. \quad \textbf{(19)}$$

Unfortunately, in the present case, we cannot find $RE\left(\hat{\lambda}_P, \hat{\lambda}_m\right)$, $RE\left(\hat{\lambda}_P, \hat{\lambda}_S\right)$, $RE\left(\hat{R}_P, \hat{R}_m\right)$, or $RE\left(\hat{R}_P, \hat{R}_S\right)$ in a nice closed form, making the analytical comparison impossible. Therefore, we obtain these risk efficiencies on the basis of simulated data using a Monte Carlo technique.

3.1 Comparison of $\hat{\lambda}_p$ with $\hat{\lambda}_m$ and $\hat{\lambda}_S$

To compare the proposed estimator $\hat{\lambda}_p$ with $\hat{\lambda}_m$ and $\hat{\lambda}_S$ given in subsection 2.1, the risk efficiencies $RE\left(\hat{R}_P, \hat{\lambda}_m\right)$ and $RE\left(\hat{\lambda}_P, \hat{\lambda}_s\right)$ are simulated on the basis of 1000 samples of size 10 each generated from (1). It may be noted that risk efficiency depends on λ, λ_0, c, k, and r besides n, which has fixed to 10 in the present study. For generation of data, different values of λ have been considered such that λ / λ_0=0.50, 1.00, 1.50, 3.00 and 4.00 where λ_0=2.00. The values of r were chosen in each case such that the sample fraction r/n=0.20(0.20)0.80. The various values of c and k considered here are c=1.50, 3.00, 6.00 and k=0.25, 0.50, 0.75. The results thus obtained are compiled in Table 1.

On the basis of Table 1, it is seen that $RE\left(\hat{\lambda}_P, \hat{\lambda}_m\right) > 1$ for almost all considered situations. It can also be seen that $RE\left(\hat{\lambda}_P, \hat{\lambda}_m\right)$ increases as λ / λ_0 increases if $c \leq 3.00$. Similarly, an increase in the value of c generally increases $RE\left(\hat{\lambda}_P, \hat{\lambda}_m\right)$ up to c=3.00; however, for c>3.00, this trend is observed only for smaller values of λ / λ_0. On the other hand, increases in the value of r/n decrease the risk efficiencies for small values of λ / λ_0 and c.

The comparison of $\hat{\lambda}_P$ with $\hat{\lambda}_s$ for different values of k shows that in some parametric space, the proposed estimator is better than the shrinkage estimator but is worse in some other situations. It may also be noted from Table 1 that as the value of c increases, the effective interval of $RE\left(\hat{\lambda}_P, \hat{\lambda}_s\right)$ increases. A similar trend is also observed for increase in the value of k. The study of the effect of variation in sample fraction r/n on $RE\left(\hat{\lambda}_P, \hat{\lambda}_s\right)$ shows that an increase in r/n increases the risk efficiencies in general if k=0.25, but for k=0.50 and 0.75 they decrease.

3.2 Comparison of estimators of R(t)

The proposed estimator $\hat{R}_P$ of R(t) and corresponding MLE and shrinkage estimator are given in (15), (16), and (17) respectively. To compare $\hat{R}_P$ with $\hat{R}_m$ and $\hat{R}_s$, the risk efficiencies RE $\left(\hat{R}_P, \hat{R}_m\right)$ and RE $\left(\hat{R}_p, \hat{R}_s\right)$ are obtained on the basis of simulated samples of size = 10 for different values of R from (10). The values of R have been chosen such that R/R_0 = 0.50(0.50)3.00 where R_0 was prefixed at 0.18. Similarly the values of r were taken such that r/n=0.20(0.20)0.80. The variation in the prior constant β_0 was taken as β_0=3.00 (3.00)12.00 with k=0.25(0.25)0.75. The results are summarized in Table 2.

It has been observed from the table that RE$\left(\hat{R}_P, \hat{R}_m\right)$>1 if R/R_0>1.00. The gains and losses due to use of $\hat{R}_P$ generally reduce as r/n increases. It may also be seen that as R/R_0 increases from 0.50, risk efficiency increases in the beginning but then starts decreasing with further increases in R/R_0. For an increase in the value of the prior constant β_0, RE$\left(\hat{R}_P, \hat{R}_m\right)$ increases for R/R_0<2.50, but a decrease is noted with large values of β_0 for R/R_0>2.50.

A comparison of the proposed estimator with the shrinkage estimator shows that RE$\left(\hat{R}_P, \hat{R}_s\right)$>1 if R/R_0>2.00. Moreover, the range of R/R_0, for which risk efficiency is greater than one, increases as k and β_0 increase except in a few cases when k=0.50. It may also be seen that an increase in the value of k increases RE$\left(\hat{R}_P, \hat{R}_s\right)$ if R/R_0<2.00. The initial increase in the value of R/R_o results in an increase in the risk efficiency, but at some values of $R/R_{o,}$ which is small for large values of β_0, a further increase causes reduction in risk efficiency. Increases in r/n do not have a uniform effect on risk efficiency. However, it may be seen that for large k and β_0 namely, k=0.75 and β_0 =12, an increase in r/n decreases the risk efficiency.

Table 1. Risk efficiency of $\hat{\lambda}_p$ with respect to $\hat{\lambda}_m$. and $\hat{\lambda}_s$ for various values of λ/λ_0 , c, and k with n=10 and λ_0 =2.00

						$RE\left(\hat{\lambda}_L, \hat{\lambda}_s\right)$											
		$RE\left(\hat{\lambda}_L, \hat{\lambda}_m\right)$				k=0.25				k=0.50				k=0.75			
C	r/n-> λ/λ_0	0.20	0.40	0.60	0.80	0.20	0.40	0.60	0.80	0.20	0.40	0.60	0.80	0.20	0.40	0.60	0.80
1.50	0.50	1.85	0.82	0.77	0.80	0.29	0.46	0.74	0.95	0.61	0.47	0.61	0.72	1.13	0.59	0.62	0.67
	1.00	2.70	0.87	0.88	0.64	0.17	0.06	0.06	0.04	0.67	0.22	0.22	0.16	1.52	0.49	0.50	0.36
	1.50	8.39	1.72	1.07	1.37	1.08	0.15	0.36	0.23	2.54	0.39	0.44	0.36	4.98	0.91	0.67	0.74
	3.00	14.50	2.95	2.20	2.06	1.40	1.32	1.15	0.83	3.09	0.83	0.64	0.46	7.46	1.38	1.00	0.87
	4.00	73.70	6.40	2.65	3.31	3.85	2.95	3.73	1.41	14.70	1.60	1.92	0.79	38.00	2.75	1.56	1.42
3.00	0.50	5.57	1.37	1.04	1.01	0.87	0.77	1.01	1.21	1.83	0.79	0.83	0.91	3.39	0.99	0.84	0.85
	1.00	18.60	2.72	1.94	1.35	1.16	0.17	0.12	0.09	4.64	0.68	0.49	0.34	10.4	1.53	1.09	0.76
	1.50	15.20	2.51	2.07	1.97	1.96	0.90	0.70	0.72	4.61	1.15	0.84	0.77	9.03	1.68	1.30	1.19
	3.00	17.50	5.04	6.43	11.60	1.69	2.26	3.28	4.63	3.73	1.42	1.84	0.57	9.02	2.34	2.89	4.9
	4.00	33.00	6.29	7.73	8.41	1.73	1.42	2.34	3.31	6.60	0.8	1.38	1.62	17.00	2.43	3.17	3.33
6.00	0.50	9.25	1.85	1.26	1.19	1.44	1.04	1.22	1.42	3.03	1.06	1.00	1.08	5.63	1.33	1.01	1.0
	1.00	89.90	8.49	4.95	2.81	5.62	0.53	0.31	0.18	22.50	2.12	1.24	0.70	50.60	4.77	2.78	1.58
	1.50	41.1	3.02	3.03	3.18	2.64	1.08	1.02	1.16	9.05	1.38	1.23	1.24	21.90	2.02	1.89	1.92
	3.00	9.53	2.07	2.31	3.69	0.91	0.93	1.17	1.47	2.03	0.58	2.66	0.82	4.90	0.96	1.04	1.55
	4.00	68.30	2.59	2.70	2.56	2.68	0.59	0.82	1.00	14.20	0.33	0.48	0.49	36.10	1.00	1.11	1.0

Table 2. Risk efficiency of $\hat{R}_p$ with respect to $\hat{R}_m$ and $\hat{R}_s$ for various values of R/R_0 , β_0 , k with n=10, R_0 =0.18, and t=3.00

						$RE(\hat{R}_L, \hat{R}_s)$											
		$RE(\hat{R}_L, \hat{R}_m)$				k=0.25				k=0.50				k=0.75			
β_0	r/n-> R/R_0	0.20	0.40	0.60	0.80	0.20	0.40	0.60	0.80	0.20	0.40	0.60	0.80	0.20	0.40	0.60	0.80
3.00	0.50	0.19	0.19	0.27	0.31	0.08	0.16	0.27	0.38	0.08	0.07	0.14	0.20	0.12	0.07	0.14	0.17
	1.00	0.03	0.76	0.84	0.85	0.06	0.05	0.05	0.05	0.26	0.19	0.21	0.21	0.58	0.43	0.47	0.48
	1.50	1.94	1.52	1.34	1.25	0.28	0.26	0.28	0.25	0.58	0.46	0.42	0.37	1.13	0.85	0.78	0.70
	2.00	3.28	2.18	1.86	1.61	1.04	0.89	1.00	1.10	1.26	0.93	0.92	0.88	2.02	1.36	1.21	1.06
	2.50	3.68	2.47	2.16	1.86	1.47	1.78	2.36	2.53	1.56	0.36	1.67	1.72	2.30	1.59	1.59	1.50
	3.00	4.21	2.53	1.94	1.74	2.85	2.41	2.77	2.88	2.43	1.01	1.80	1.78	2.88	1.85	1.53	1.40
6.00	0.50	0.31	0.24	0.33	0.35	0.14	0.22	0.33	0.43	0.13	0.09	0.16	0.22	0.19	0.1	0.16	0.20
	1.00	2.22	1.36	1.32	1.24	0.14	0.08	0.08	0.08	0.56	0.34	0.33	0.31	1.25	0.76	0.75	0.70
	1.50	5.51	3.16	2.37	1.96	0.82	0.54	0.48	0.38	1.67	0.95	0.74	0.58	3.23	1.82	1.37	1.11
	2.00	8.22	4.03	2.95	2.33	2.43	1.64	1.98	1.59	3.06	1.71	1.46	1.22	4.99	2.50	1.91	1.13
	2.50	5.74	3.75	2.72	2.21	2.26	2.73	3.06	3.08	2.40	2.08	2.13	2.07	3.55	2.42	2.02	1.78
	3.00	4.03	2.74	2.09	1.90	2.73	2.58	2.93	3.14	2.34	1.92	1.90	1.92	2.78	1.97	1.62	1.51
9.00	0.50	0.28	0.29	0.37	0.22	0.18	0.25	0.37	0.54	0.14	0.11	0.18	0.25	0.17	0.12	0.18	0.18
	1.00	3.85	2.18	1.91	1.38	0.24	0.13	0.12	0.09	0.96	0.53	0.48	0.34	2.17	1.19	1.08	0.77
	1.50	10.59	5.54	3.70	3.11	1.13	0.96	0.75	0.63	2.73	1.67	1.15	1.04	5.88	3.19	2.14	1.87
	2.00	10.47	5.24	3.69	2.45	3.12	2.14	1.98	1.76	3.93	2.23	1.82	2.65	6.38	3.26	2.39	2.55
	2.50	5.52	3.64	2.54	2.11	2.20	2.62	2.82	2.88	2.33	2.01	1.20	1.96	3.54	2.35	1.9	1.70
	3.00	3.10	2.35	1.70	1.72	2.10	2.23	2.57	2.85	1.81	1.67	1.67	1.76	2.14	1.71	1.42	1.38
12.00	0.50	0.34	0.32	0.39	0.24	0.22	0.28	0.40	0.46	0.17	0.12	0.20	0.25	0.21	0.13	0.20	0.18
	1.00	5.93	0.04	2.61	2.09	0.37	0.19	0.16	0.10	1.48	0.76	0.65	0.62	3.33	1.71	1.47	1.43
	1.50	19.28	8.48	5.25	3.69	0.06	1.46	1.07	1.10	4.97	2.55	1.64	0.47	10.07	4.89	3.03	2.01
	2.00	9.71	5.53	3.94	2.58	2.88	2.26	2.11	2.09	3.64	2.35	1.95	1.23	5.92	3.44	2.56	1.78
	2.50	4.90	3.13	2.21	1.88	1.96	2.28	2.48	2.61	1.06	1.74	1.73	1.75	3.04	2.01	1.74	1.51
	3.00	2.59	2.03	1.57	1.51	1.73	1.91	2.21	2.51	1.48	1.42	1.43	1.54	1.76	1.46	1.22	1.21

4 CONCLUSION

We have seen that the proposed estimator $\hat{\lambda}_P$ performs better than MLE over a wide range of parameter space. Thus, $\hat{\lambda}_p$ may be recommended as an alternative estimator against MLE with large c(≥ 1.50) , provided λ is not very small as compared to λ_0. It has also been observed that $\hat{\lambda}_P$ is superior to the shrinkage estimator $\hat{\lambda}_s$ for k=0.75 for large values of c. For small k, i.e., k=0.25 and k=0.50, however, the proposed estimator $\hat{\lambda}_p$ may still be preferred to $\hat{\lambda}_s$ if the true value of λ is greater than the guessed value λ_0. In brief we may conclude that the proposed estimator can be safely used with large values of c when the guessed estimate is an underestimate of true value. It has also been seen that the proposed estimator of reliability is always more efficient than the corresponding MLE if estimate R_0 is less than the true value of parameter R=R(t).

On the other hand, comparing the estimator $\hat{R}_p$ with shrinkage estimator $\hat{R}_s$, it is noticed that if R_0 is an underestimate of its true value, with the risk of $\hat{R}_p$ generally less than that of $\hat{R}_s$ for k=0.75, the region of gain increases for large values of β_0. However, if k=0.25 and 0.75, $RE\left(\hat{R}_p, \hat{R}_s\right) > 1$ if R/R_0 is large. Thus, the proposed estimator $\hat{R}_p$ may be recommended for use with large β_0 only if R_0 is expected to be smaller than its true value.

5 REFERENCES

Canfield, R.V. (1970) A Bayesian approach to reliability estimation using a loss function. *IEEE Trans. Reliab*. 19, 13-16.

Lemmer, H.H. (1981) From ordinary to Bayesian shrinkage estimators. *South African Statist. Jr*., (15), 57-72.

Martz, F.H. & Waller, R.A. (1982) *Bayesian Reliability Analysis*. Wiley: New York.

Mehta, J.S. & Srinivasan, R. (1971) Estimation of the mean by shrinkage to a point. *JASA*, 66, 86-90.

Pandey, B.N. (1979) On shrinkage estimation of normal population variance, *Commun. Statist*. A8 (4) 359-365.

Pandey, B.N. & Srivastava, R. (1985) On shrinkage estimation of the exponential scale parameters. *IEEE. Trans. Relb*., R-34, 224-226.

Pandey, M. & Upadhyay, S. (1985) Bayesian shrinkage estimation of reliability from censored samples with exponential failure model. *South African Statist. Jr*., (19) 21-33.

Pandey, M. & Upadhyay, S. (1988) Bayesian inference in mixture of two exponentials. *Microelcton Relab*., 28, 217-221.

Raiffa, H. & Schlaifer, R. (1961) *Applied statistical Decision Theory*. Harvard University: Boston, MA.

Sinha, *S.K.* (1985) *Reliability and Life Testing*. Wiley: New Delhi.

Thompson, J. R. (1968) Some shrinkage techniques for estimating the means. *JASA,* 67, 113-123.