BAYESIAN ESTIMATION IN THE PROPORTIONAL HAZARDS MODEL OF RANDOM CENSORSHIP UNDER ASYMMETRIC LOSS FUNCTIONS

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ABSTRACT

In this paper, we consider the Bayesian estimation of parameters in the proportional hazards model of random censorship for the Weibull distribution under different asymmetric loss functions. It is well-known for the Weibull distribution that a joint conjugate prior on the parameters does not exist; we use both the informative and noninformative priors on the model parameters. Bayes estimates under LINEX and general entropy loss functions are obtained using the Gibbs sampling scheme. A simulation study is carried out to observe the behavior of the proposed estimators for different sample sizes and for different censoring parameters. It is observed that the Bayes estimators under LINEX and general entropy loss functions can be used effectively with the appropriate choice of respective loss function parameters. One real data set is analyzed for illustrative purposes.

Keywords: Random censoring, Bayes estimators, Proportional hazards model, Gibbs sampling, LINEX loss function, General entropy loss function, Markov chain Monte Carlo

1 INTRODUCTION

Censoring is an unavoidable feature of reliability and life-testing experiments. In these experiments the objects under study are lost or removed from the test before failure, and the event of interest may not always be observed for all the objects in the experiment. Clearly, to wait for the last observation would not be possible as it is necessary to report the results from the study as soon as possible. Among the different types of right censoring, type I and type II are the most popular censoring schemes. In these censoring schemes, objects are removed from the test at the termination of the test. In situations where it is desirable to remove the objects from the test other than at the final termination point, the random censoring scheme provides a suitable mechanism. The random censoring scheme is an important type of right censoring in which the time of censoring is not fixed but taken as random. The type I right censoring scheme can be considered as a special case of random censoring scheme in which censoring takes place at some fixed time point. The random censoring scheme has not been given much consideration in the Bayesian context. Some of the work on this censoring scheme can be found in Sarhan (2003), Liang (2004), Abu-Taleb, Smadi, and Alawneh (2007), Friesl and Hurt (2007), and Sarhan and Abuammoh (2008). These authors focused on the exponential distribution, which is appropriate only when the hazard rate is constant. There are many phenomena in life testing experiments where the suitability of the exponential distribution is questioned. For situations where the hazard rate is increasing or decreasing, the generalized exponential (GE), gamma, and Weibull distributions are most suitable. The scale and shape parameters of these distributions make them flexible for analyzing any general life time data. The Weibull distribution has an extra edge over the gamma distribution in the sense that its distribution and hazard functions can be expressed in closed-forms, which are not possible for gamma distribution when the shape parameter is not an integer. An extensive literature is available on the statistical inferences of the unknown parameters of the life time distributions both in classical and Bayesian setups. Gupta and Kundu (2006) compared the Weibull and generalized exponential distributions for Fisher information. They noted that although both the distributions fit the data equally well, the corresponding Fisher information matrices are quite different. Kundu (2008) dealt with the Bayesian inference of the Weibull distribution under a progressive censoring scheme and provided a methodology to compare two different censoring schemes. Raqab and Madi (2009) investigated the properties of the two parameter exponentiated Rayleigh distribution using Gibbs and Metropolis samplers. Wahed (2006) discussed the Burr type XII distribution for Bayesian estimation of parameters under symmetric and asymmetric loss functions. Singh et al. (2008) approximated the Bayes estimates of generalized exponential distribution under linear exponential loss function using Lindley's approximation. However, not much work has been done on the statistical inferences of the unknown parameters of the life time distributions under the random censoring scheme in the Bayesian framework.

Mainly, two models of random censorship are considered to analyze the data resulting from the random censoring scheme. In the general random censorship model, it is assumed that the survival time variable and censoring time variable follow the same distribution and that they are independent. In the proportional hazards (PH) model, it is further assumed that the hazard functions of the survival time and censoring time variables are proportional. For detail on these models, see Koziol and Green (1976), Hollander and Peña (1989), Hurt (1992), and Csörgő and Faraway (1998). In this paper, we consider the Weibull distribution under the PH model of random censorship for Bayesian estimation of parameters. Bayesian analysis requires appropriate loss function and priors for the model parameters in addition to the experimental data. There is no clear cut way in which one can say that one prior is better than the other; the important thing is the relationship between the prior distribution and the loss function. Since a joint conjugate prior for the parameters does not exist, we consider both the informative and noninformative priors. However, when sufficient information about the parameters is available, it is better to use informative priors than vague priors. In order to select the best decision in decision theory, an appropriate loss function must be specified. A squared error (SE) loss function is generally used for this purpose. The use of the SE loss function is well justified when the loss is symmetric with respect to overestimation and underestimation. In situations where the loss due to overestimation is more serious than the loss due to underestimation and vice-versa, the asymmetric loss functions are more suitable, as pointed out by several authors, see for example, Varian (1975), Basu and Ebrahimi (1991), and Calabria and Pulcini (1996). The aim of the present paper is to obtain the Bayes estimators of unknown parameters and compare them under LINEX and general entropy loss functions using different proportions of censored observations.

The rest of the paper is organized as follows. Section 2 contains the model and assumptions. In Section 3, we discuss asymmetric loss functions, informative and noninformative priors, the Gibbs sampling scheme, and Bayes estimators. A simulation study is considered in Section 4. A real data set is analyzed in Section 5, and we conclude the paper in Section 6.

2 THE MODEL AND ASSUMPTIONS

Let $X_1, X_2, ..., X_n$ be independent and identically distributed random variables with distribution function F(x) and density function f(x). Let $T_1, T_2, ..., T_n$ also be independent and identically distributed random variables with distribution function G(t) and density function g(t). In the context of reliability and survival analysis, X_i is the true survival time of an individual censored by T_i from the right. The experiment thus results in observing pairs $(Y_1, D_1), (Y_2, D_2), ..., (Y_n, D_n)$, where $Y_i = Min(X_i, T_i)$ and $D_i = \mathrm{I}(X_i \le T_i)$, indicating whether the survival time X_i or the censoring time T_i has occurred first, for i = 1, 2, . . . , n. In the random censorship model it is assumed that X_i and T_i are independent. With these assumptions, it is simple to show that the joint density function of Y_i and D_i is

$$f_{Y_i,D_i}(y_i, d_i) = \{f_X(y_i)(1 - G_T(y_i))\}^{d_i} \{g_T(y_i)(1 - F_X(y_i))\}^{1-d_i} ; y_i \ge 0, d_i = 0,1 \quad (1)$$

Under the PH model, the distributions of X_i and T_i are related by

$$1 - G_T(y) = \{1 - F_X(y)\}^{\beta}, \quad (2)$$

for some $\beta > 0$. For $\beta = 0$, expression (2) represents the case of no censoring. From (1) and (2), it can be shown that

$$f_{Y_i,D_i}(y_i, d_i) = f_X(y_i)\{1 - F_X(y_i)\}^{\beta} \beta^{1-d_i}; \quad y_i \ge 0, d_i = 0,1 \quad (3)$$

We assume that X_i has a Weibull distribution with scale parameter λ and shape parameter θ. The probability density function and the distribution function of the Weibull distribution are

$$f_X(x_i; \theta, \lambda) = \theta\lambda x_i^{\theta-1} e^{-\lambda x_i^{\theta}} \quad ; \quad x_i > 0, \ \theta, \lambda > 0 \quad (4)$$

$$F_X(x_i; \theta, \lambda) = 1 - e^{-\lambda x_i^{\theta}} \quad x_i > 0, \ \theta, \lambda > 0 \quad (5)$$

Using (4) and (5), we can express (3) as

$$f_{Y_i,D_i}\left(y_i,d_i;\theta,\lambda,\beta\right)=\theta\lambda y_i^{\theta-1}e^{-(1+\beta)\lambda y_i^{\theta}}\beta^{1-d_i};\quad y_i,\theta,\lambda,\beta>0,d_i=0,1 \tag{6}$$

The marginal distributions of Y_i and D_i can be obtained from (6) as

$$f_{Y_i}\left(y_i;\theta,\lambda,\beta\right)=\left(1+\beta\right)\theta\lambda y_i^{\theta-1}e^{-(1+\beta)\lambda y_i^{\theta}}$$

and $f_{D_i}\left(d_i;p\right)=p^{d_i}\left(1-p\right)^{1-d_i};\quad p\in(0,1),$ respectively, where $p=P\left(X_i\le T_i\right)=\dfrac{1}{1+\beta}.$

For a random sample $\left(Y,D\right)=\left(\left(Y_1,D_1\right),\left(Y_2,D_2\right),...,\left(Y_n,D_n\right)\right)$ of size n from (6), the likelihood function is

$$l\left(\theta,\lambda,\beta\middle|\left(Y,D\right)\right)=\theta^n\lambda^n\prod_{i=1}^{n}y_i^{\theta-1}e^{-(1+\beta)\lambda\sum_{i=1}^{n}y_i^{\theta}}\beta^{n-\sum_{i=1}^{n}d_i} \tag{7}$$

3 BAYESIAN ESTIMATION OF PARAMETERS

In this section, we discuss the loss functions and the priors to be used to obtain the Bayes estimates of parameters.

3.1 Loss Functions

The Bayesian estimation requires the choice of a suitable loss function in order to obtain the estimator that has minimum loss among the other possible estimators. The SE loss function is generally used for this purpose. The applicability of the SE loss function is motivated by the fact that the Bayes estimator under this loss function is simply the posterior mean. The use of the SE loss function is well justified when over estimation and under estimation of equal magnitude have the same consequences. When the true loss is not symmetric with respect to over estimation and under estimation, the asymmetric loss functions are used to represent the consequences of different errors. A very useful asymmetric loss function is LINEX (linear exponential) introduced by Varian (1975). The LINEX loss function with parameters k and c is defined as

$$L\left(\Delta\right)=L\left(\hat{\theta}_{LE},\theta\right)=k\left(e^{c\Delta}-c\Delta-1\right)\quad;\ c\neq0\quad, \tag{8}$$

where $\Delta=\hat{\theta}_{LE}-\theta$ and $\hat{\theta}_{LE}$ is a decision rule to estimate parameter θ. The sign of parameter c represents the direction of asymmetry, and its magnitude reflects the degree of asymmetry. For $c<0$, the underestimation is more serious than the overestimation, and for $c>0$, the overestimation is more serious than the underestimation. For c close to zero, the LINEX loss function is approximately the squared error loss function. The Bayes estimate under the LINEX loss function is

$$\hat{\theta}_{LE}=-\frac{1}{c}\ln E\left(e^{-c\theta}\right), \tag{9}$$

where E denotes the posterior expectation. Another suitable asymmetric loss function is the general entropy (GE) loss function proposed by Calabria and Pulcini (1996). This loss function is a generalization of the entropy loss used by Dey, Ghosh, and Srinivasan (1987) and Dey and Liu (1992). The GE loss function with parameters q and k_1 is defined as

$$L\left(\hat{\theta}_{GE},\theta\right)=k_1\left[\left(\frac{\hat{\theta}_{GE}}{\theta}\right)^q-q\ln\left(\frac{\hat{\theta}_{GE}}{\theta}\right)-1\right]\quad;\ q\neq0\quad, \tag{10}$$

where $\hat{\theta}_{GE}$ is the decision rule to estimate parameter θ. For $q>0$, a positive error has a more serious effect than a negative error, and for $q<0$, a negative error has a more serious effect than a positive error. The Bayes estimate under the GELF is

$$\hat{\theta}_{GE}=\left[E\left(\theta^{-q}\right)\right]^{-\frac{1}{q}}. \tag{11}$$

Note that for $q=-1$, the Bayes estimate coincides with the Bayes estimate under the SE loss function and for $q=1$, the Bayes estimate coincides with the Bayes estimate under the weighted squared error loss function

$$L\left(\hat{\theta}_{WSE},\theta\right)=\frac{\left(\theta-\hat{\theta}_{WSE}\right)^2}{\theta},$$

where $\hat{\theta}_{WSE}$ is the decision rule to estimate parameter θ.

3.2 Informative Prior

We assume the following independent gamma priors on the model parameters

$$\begin{aligned}\pi_1(\theta)&=\frac{b_1^{a_1}}{\Gamma(a_1)}\theta^{a_1-1}e^{-b_1\theta}; \qquad a_1,b_1,\theta>0\\ \pi_2(\lambda)&=\frac{b_2^{a_2}}{\Gamma(a_2)}\lambda^{a_2-1}e^{-b_2\lambda}; \qquad a_2,b_2,\lambda>0\\ \pi_3(\beta)&=\frac{b_3^{a_3}}{\Gamma(a_3)}\beta^{a_3-1}e^{-b_3\beta}; \qquad a_3,b_3,\beta>0\end{aligned} \tag{12}$$

The reason for using gamma priors is the flexibility of gamma distributions that include the noninformative priors (uniform) on the scale and the shape parameters as special cases. Many authors have used independent gamma priors on the scale and the shape parameters of Weibull distribution under different censoring schemes, see for example, Kundu (2008), Wahed (2006), and Joarder, Krishna, and Kundu (2011).

The joint prior distribution of the unknown parameters, from (12), is

$$\pi(\theta,\lambda,\beta)\propto\theta^{a_1-1}e^{-b_1\theta}\lambda^{a_2-1}e^{-b_2\lambda}\beta^{a_3-1}e^{-b_3\beta} \tag{13}$$

Combining (7) and (12), the joint posterior density function of θ, λ, and β given data is

$$\pi\left(\theta,\lambda,\beta\middle|(y,d)\right)\propto\theta^{n+a_1-1}e^{-\theta\left\{b_1-\sum_{i=1}^{n}\ln y_i\right\}}\lambda^{n+a_2-1}e^{-\lambda\left(b_2+(1+\beta)\sum_{i=1}^{n}y_i^{\theta}\right)}\beta^{n-\sum_{i=1}^{n}d_i+a_3-1}e^{-\beta b_3} \tag{14}$$

The posterior expectation of any function of parameters, say $U(\theta,\lambda,\beta)$, can be written as

$$E\left(U(\theta,\lambda,\beta)\middle|(y,d)\right)$$

$$=\frac{\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}U(\theta,\lambda,\beta)\theta^{n+a_1-1}e^{-\theta\left\{b_1-\sum_{i=1}^{n}\ln y_i\right\}}\lambda^{n+a_2-1}e^{-\lambda\left(b_2+(1+\beta)\sum_{i=1}^{n}y_i^{\theta}\right)}\beta^{n-\sum_{i=1}^{n}d_i+a_3-1}e^{-\beta b_3}\,d\theta\,d\lambda\,d\beta}{\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\theta^{n+a_1-1}e^{-\theta\left\{b_1-\sum_{i=1}^{n}\ln y_i\right\}}\lambda^{n+a_2-1}e^{-\lambda\left(b_2+(1+\beta)\sum_{i=1}^{n}y_i^{\theta}\right)}\beta^{n-\sum_{i=1}^{n}d_i+a_3-1}e^{-\beta b_3}\,d\theta\,d\lambda\,d\beta} \tag{15}$$

However, it is not possible to evaluate (15) in closed-form. We use the Gibbs sampler to generate (θ,λ,β) from posterior distribution (14). Once we have a mechanism to generate samples from (14), we can use these samples to obtain the approximate Bayes estimates under different loss functions.

3.3 Gibbs Sampling

From (14), the full conditionals for θ, λ, and β can be obtained up to proportionality as

$$\pi_1\left(\theta\middle|\lambda,\beta,(y,d)\right)\propto\theta^{n+a_1-1}e^{-\theta\left(b_1-\sum_{i=1}^{n}\ln y_i\right)}e^{-(1+\beta)\lambda\sum_{i=1}^{n}y_i^{\theta}} \tag{16}$$

$$\pi_2\left(\lambda\middle|\theta,\beta,(y,d)\right)\propto\lambda^{n+a_2-1}e^{-\lambda\left(b_2+(1+\beta)\sum_{i=1}^{n}y_i^{\theta}\right)} \tag{17}$$

$$\pi_3\left(\beta\left|\theta,\lambda,(y,d)\right.\right)\propto\beta^{n-\sum_{i=1}^{n}d_i+a_3-1}e^{-\beta\left(b_3+\lambda\sum_{i=1}^{n}y_i^{\theta}\right)} \tag{18}$$

The full conditional forms (17) and (18) are the gamma densities, so the samples of λ and β can be easily generated using any of the gamma generating routines. The full conditional form (16) is log-concave since

$$\frac{\partial^2\left(\pi_1\left(\theta\left|\lambda,\beta,(y,d)\right.\right)\right)}{\partial\theta^2}=\frac{-(n+a_1-1)}{\theta^2}-(1+\beta)\lambda\sum_{i=1}^{n}y_i^{\theta}(\ln y_i)^2<0$$

Thus the samples of θ can be generated using the method suggested in Devroye (1984). Now following the idea given in Geman and Geman (1984) and using (16), (17), (18), it is possible to generate samples from posterior distribution (14). Starting with suitable initial values of parameters, say $(\theta_0,\lambda_0,\beta_0)$, the following procedure can be used to generate (θ,λ,β)

Step 1. Generate θ_1 from the log-concave density (16) using the method suggested in Devroye (1984)

Step 2. Generate λ_1 from gamma$\left(n+a_2, b_2+(1+\beta_0)\sum_{i=1}^{n}y_i^{\theta_1}\right)$

Step 3. Generate β_1 from gamma$\left(n-\sum_{i=1}^{n}d_i+a_3, b_3+\lambda_1\sum_{i=1}^{n}y_i^{\theta_1}\right)$

Step 4. Repeat steps 1, 2, and 3 M times to obtain $(\lambda_1,\theta_1,\beta_1),\ldots,(\lambda_M,\theta_M,\beta_M)$ (19)

3.4 Bayes Estimates under LINEX Loss Function

Using the generated sample of θ in (19), the approximate value of $E\left(e^{-c_1\theta}\right)$ in (9) can be obtained from

$$\frac{\sum_{j=1}^{M}e^{-c_1\theta_j}}{M}$$

Thus the approximate Bayes estimate of parameter θ under the LINEX loss function is

$$\hat{\theta}_{LE}=-\frac{1}{c_1}\ln\left(\frac{\sum_{j=1}^{M}e^{-c_1\theta_j}}{M}\right)$$

Similarly, the approximate Bayes estimates of λ and β under the LINEX loss function are

$$\hat{\lambda}_{LE}=-\frac{1}{c_2}\ln\left(\frac{\sum_{j=1}^{M}e^{-c_2\lambda_j}}{M}\right)\quad\text{and}\quad\hat{\beta}_{LE}=-\frac{1}{c_3}\ln\left(\frac{\sum_{j=1}^{M}e^{-c_3\beta_j}}{M}\right)$$

3.5 Bayes Estimates under General Entropy Loss Function

Using the generated sample of θ in (19), the approximate value of $E\left(\theta^{-q_1}\right)$ in (11) can be obtained from

$\frac{\sum_{j=1}^{M}\theta_j^{-q_1}}{M}$. Thus the approximate Bayes estimate of parameter θ under GE is $\hat{\theta}_{GE}=\left[\frac{\sum_{j=1}^{M}\theta_j^{-q_1}}{M}\right]^{-\frac{1}{q_1}}$

Similarly, the Bayes estimates of parameters θ and λ under general entropy loss function are

$$\hat{\lambda}_{GE}=\left[\frac{\sum_{j=1}^{M}\lambda_j^{-q_2}}{M}\right]^{-\frac{1}{q_2}} \quad \text{and} \quad \hat{\beta}_{GE}=\left[\frac{\sum_{j=1}^{M}\beta_j^{-q_3}}{M}\right]^{-\frac{1}{q_3}}$$

3.6 Jeffreys Prior

The determinant of the Fisher information matrix can be shown as $\frac{n^3\pi^2}{6\theta^2\lambda^2\beta(1+\beta)^2}$. Therefore, the Jeffreys prior can be written as

$$\pi_J(\theta,\lambda,\beta)\propto\left|Det\left(I(\theta')\right)\right|^{\frac{1}{2}}\propto\frac{1}{\theta\lambda\beta^{\frac{1}{2}}(1+\beta)} \tag{20}$$

Multiplying the likelihood function in (7) and the Jeffreys prior in (20), the joint posterior distribution of θ, λ, and β given data is

$$\pi_J\left(\theta,\lambda,\beta\middle|(y,d)\right)\propto\theta^{n-1}e^{\theta\sum_{i=1}^{n}\ln y_i}\lambda^{n-1}e^{-\lambda(1+\beta)\sum_{i=1}^{n}y_i^{\theta}}\frac{\beta^{n-\sum_{i=1}^{n}d_i-\frac{1}{2}}}{1+\beta} \tag{21}$$

Thus the posterior expectation of any function of parameters, say $U(\theta,\lambda,\beta)$, is

$$E\left(U(\theta,\lambda,\beta)\middle|(y,d)\right)=\frac{\int_0^\infty\int_0^\infty\int_0^\infty U(\theta,\lambda,\beta)\theta^{n-1}e^{\theta\sum_{i=1}^{n}\ln y_i}\lambda^{n-1}e^{-\lambda(1+\beta)\sum_{i=1}^{n}y_i^{\theta}}\frac{\beta^{n-\sum_{i=1}^{n}d_i-\frac{1}{2}}}{1+\beta}\,d\theta\,d\lambda\,d\beta}{\int_0^\infty\int_0^\infty\int_0^\infty\theta^{n-1}e^{\theta\sum_{i=1}^{n}\ln y_i}\lambda^{n-1}e^{-\lambda(1+\beta)\sum_{i=1}^{n}y_i^{\theta}}\frac{\beta^{n-\sum_{i=1}^{n}d_i-\frac{1}{2}}}{1+\beta}\,d\theta\,d\lambda\,d\beta} \tag{22}$$

However, it is not possible to evaluate (22) analytically. We use the Gibbs sampling scheme to obtain the Bayes estimates of parameters.

3.7 Gibbs Sampling

We have already used the Gibbs sampling scheme in Section 3.3 to generate samples from the full conditionals of θ, λ, and β. In this section, we use the Gibbs sampling scheme to generate samples directly from the joint posterior distribution of θ, λ, and β. To generate Gibbs samples directly from the joint posterior distribution is more natural (if possible) than to generate Gibbs samples from the full conditional forms. We need the following results to proceed further.

Theorem: (a) The conditional posterior distribution of λ given θ, β, and data is

$$\text{Gamma}\left(n,(1+\beta)\sum_{i=1}^{n}y_i^{\theta}\right) \tag{23}$$

(b) The marginal posterior distribution of β given data is

$$\text{BetaII}\left(n-\sum_{i=1}^{n}d_i+\frac{1}{2},\sum_{i=1}^{n}d_i+\frac{1}{2}\right) \tag{24}$$

(c) The marginal posterior distribution of θ given data is

$$g\left(\theta\middle|(y,d)\right)\propto\theta^{n-1}e^{\theta\sum_{i=1}^{n}\ln y_i}\left(\sum_{i=1}^{n}y_i^{\theta}\right)^{-n} \tag{25}$$

and (d) $g\left(\theta\middle|(y,d)\right)$ is log-concave,

where BetaII denotes the beta distribution of second kind.

Proof: Parts (a), (b) and (c) are trivial so are not provided, and for part (d) see Appendix A.

Now, following the ideas of Geman and Geman (1984) and using the above Theorem, we suggest the following procedure to generate (θ, λ, β) from the posterior distribution (21).

Step 1. Generate θ_1 from the log-concave density (25) using the method proposed in Devroye (1984).

Step 2. Generate β_1 from the BetaII $\left(n-\sum_{i=1}^{n}d_i+\frac{1}{2},\sum_{i=1}^{n}d_i+\frac{1}{2}\right)$

Step 3. Generate λ_1 from Gamma $\left(n,(1+\beta_1)\sum_{i=1}^{n}y_i^{\theta_1}\right)$

Step 4. Repeat steps 1, 2, and 3 M times to obtain $(\lambda_1,\theta_1,\beta_1),\ldots,(\lambda_M,\theta_M,\beta_M)$

Now the Bayes estimates under LINEX and general entropy loss functions can be easily obtained using the generated samples in Step 4 as in Sections 3.4 and 3.5.

4 SIMULATION

In this section, we perform a Monte Carlo simulation to observe the behavior of the proposed estimators of the parameters for different sample sizes, for different priors, for different loss function parameters (LFPs), and for different proportions of uncensored observations. We consider different sample sizes: n = 20, 40, 60; different proportions of uncensored observations: p = 0.50, 0.75, 0.90; different values of the loss function parameter: c = q = -1.5, -0.9, -0.3, 0.3, 0.9, 1.5, 2.1; different sets of model parameter values: θ = 1.5, λ = 1, β = 1; θ = 1.5, λ = 1, β =0.3333; θ = 1.5, λ = 1, β = 0.1111, and different combinations of hyperparameters: a_1 = 0, b_1 = 0, a_2 = 0, b_2 = 0, a_3 = 0, b_3 = 0 (prior1) and a_1 = 3, b_1 = 2, a_2 = 1, b_2 = 1, a_3 = 1, b_3 = 1 (prior2) when θ = 1.5, λ = 1, β = 1; prior1 and a_1 = 3, b_1 = 2, a_2 = 1, b_2 = 1, a_3 = 1, b_3 = 3 (prior2) when θ = 1.5, λ = 1, β = 0.3333; prior1 and a_1 = 3, b_1 = 2, a_2 = 1, b_2 = 1, a_3 = 1, b_3 = 9 (prior2) when θ = 1.5, λ = 1, β = 0.1111. Note that in prior2, the hyperparameters are taken so that the means of the gamma priors are the same as the original means. Moreover, when all the hyperparameters are zero (prior1), the gamma priors reduce to the uniform noninformative priors on the scale and shape parameters. Prior2 represents the informative gamma priors with means equal to the corresponding parameter values. Prior1 and prior2 are the two extreme cases. On one extreme, we are completely uninformed about the parameters, and on other extreme, we know the parameters completely. For a particular case, we generate 1000 randomly censored samples from (6) and for each sample we compute the Bayes estimates under LINEX and GE loss functions based on 20,000 MCMC samples with 10,000 samples as the burn-in period. First, we examine the trace plots for a number of samples to determine the required number of iterations that should be run to have convergence. Then, we check the Monte Carlo error for each parameter to determine the required number of iterations that should be run to have reasonable accuracy. The averages and mean square errors (MSEs) of Bayes estimators are obtained from these 1000 replications. The results are reported in Tables 1-3 (MSE1 and MSE2 denote the MSEs of parameters under LINEX and GE loss functions respectively). It is clear from these results that as the sample size increases, the MSEs of Bayes estimators under both the loss functions decrease no matter what proportion of censored observations is used. This rate of decrease in MSEs is higher for informative priors as compared to noninformative priors. This means that if one has relevant information about the parameters, it is better to use informative priors than noninformative priors. It can also be noted that the rate of decrease in MSEs is significantly higher for small to moderate sample sizes as compared to moderate to large sample sizes. This means that it is better economically to use moderate sample sizes than large sample sizes. It is also observed that the effect of censored observations on the MSEs is insignificant. The suitable range for loss function parameters *c* and *q* varies with the Bayes estimators of model parameters θ, λ, and β. This range for the Bayes estimators of shape θ is $(1.5 < c = q < 2.5)$ and for the Bayes estimators of scale parameter λ is $(0.9 < c < 2.1)$ $(0.3 < q < 0.9)$. However, the range of suitable values of *c* and *q* for the Bayes estimators of censoring parameter β varies with the proportion of censored observations. In their respective suitable ranges, the performance of the Bayes estimators under LINEX and GE loss functions is approximately equal in terms of corresponding MSEs. The results indicate that the Bayes estimators under LINEX and GE loss functions can be used effectively with the appropriate choice of respective LFPs instead of the Bayes estimators under the traditional SE loss function (*c* close to zero and *q* = -1).

Table 1. Average values of the Bayes estimators of θ and the corresponding MSEs

(a)

p = 0.50		Prior1				Prior2			
n	c = q	$\hat{\theta}_{LE}$	$\hat{\theta}_{GE}$	MSE1	MSE2	$\hat{\theta}_{LE}$	$\hat{\theta}_{GE}$	MSE1	MSE2
20	-1.5	1.6272	1.6130	0.1445	0.1114	1.6394	1.5919	0.1072	0.0838
	-0.9	1.6197	1.5978	0.1276	0.1058	1.6153	1.5782	0.0952	0.0797
	-0.3	1.5932	1.5825	0.1131	0.1007	1.5919	1.5644	0.0849	0.0759
	0.3	1.5677	1.5670	0.1008	0.0960	1.5693	1.5505	0.0761	0.0725
	0.9	1.5429	1.5514	0.0905	0.0918	1.5473	1.5365	0.0689	0.0695
	1.5	1.5190	1.5357	0.0820	0.0880	1.5260	1.5223	0.0629	0.0668
	2.1	1.4947	1.5124	0.0735	0.0837	1.5087	1.5061	0.0597	0.0632
40	-1.5	1.5740	1.5507	0.0478	0.0414	1.5677	1.5457	0.0425	0.0369
	-0.9	1.5622	1.5435	0.0445	0.0401	1.5565	1.5388	0.0396	0.0358
	-0.3	1.5505	1.5363	0.0416	0.0389	1.5454	1.5319	0.0371	0.0347
	0.3	1.5390	1.5290	0.0391	0.0378	1.5345	1.5249	0.0349	0.0337
	0.9	1.5277	1.5216	0.0368	0.0368	1.5238	1.5180	0.0329	0.0329
	1.5	1.5165	1.5143	0.0349	0.0359	1.5132	1.5110	0.0312	0.0321
	2.1	1.5053	1.5037	0.0326	0.0347	1.5026	1.5019	0.0309	0.0318
60	-1.5	1.5454	1.5310	0.0280	0.0254	1.5454	1.5310	0.0280	0.0254
	-0.9	1.5381	1.5264	0.0267	0.0248	1.5381	1.5264	0.0267	0.0248
	-0.3	1.5308	1.5217	0.0255	0.0243	1.5308	1.5217	0.0255	0.0243
	0.3	1.5236	1.5172	0.0243	0.0237	1.5236	1.5172	0.0243	0.0237
	0.9	1.5165	1.5126	0.0233	0.0233	1.5165	1.5126	0.0233	0.0233
	1.5	1.5094	1.5079	0.0225	0.0229	1.5094	1.5079	0.0225	0.0229
	2.1	1.5043	1.5031	0.0221	0.0225	1.5023	1.5008	0.0221	0.0224

(b)

p = 0.75		Prior1				Prior2			
n	c = q	$\hat{\theta}_{LE}$	$\hat{\theta}_{GE}$	MSE1	MSE2	$\hat{\theta}_{LE}$	$\hat{\theta}_{GE}$	MSE1	MSE2
20	-1.5	1.6489	1.5956	0.1480	0.1154	1.6241	1.5771	0.1119	0.0886
	-0.9	1.6218	1.5805	0.1315	0.1102	1.6002	1.5635	0.1000	0.0847
	-0.3	1.5957	1.5653	0.1173	0.1055	1.5771	1.5497	0.0899	0.0812
	0.3	1.5706	1.5500	0.1054	0.1012	1.5547	1.5259	0.0813	0.0781
	0.9	1.5462	1.5345	0.0954	0.0973	1.5329	1.5219	0.0741	0.0753
	1.5	1.5226	1.5189	0.0871	0.0940	1.5119	1.5078	0.0683	0.0729

	2.1	1.4989	1.5033	0.0788	0.0908	1.4962	1.4937	0.0625	0.0708
40	-1.5	1.5554	1.5425	0.0502	0.0443	1.5601	1.5385	0.0448	0.0397
	-0.9	1.5537	1.5353	0.0472	0.0432	1.5491	1.5316	0.0422	0.0387
	-0.3	1.5422	1.5280	0.0446	0.0422	1.5382	1.5248	0.0399	0.0379
	0.3	1.5308	1.5208	0.0423	0.0413	1.5274	1.5178	0.0379	0.0371
	0.9	1.5197	1.5135	0.0403	0.0405	1.5168	1.5109	0.0362	0.0364
	1.5	1.5087	1.5062	0.0386	0.0399	1.5064	1.5039	0.0347	0.0358
	2.1	1.4977	1.4989	0.0371	0.0394	1.4979	1.4969	0.0334	0.0353
60	-1.5	1.5370	1.5225	0.0283	0.0260	1.5348	1.5208	0.0264	0.0243
	-0.9	1.5296	1.5178	0.0272	0.0256	1.5276	1.5162	0.0253	0.0239
	-0.3	1.5223	1.5131	0.0262	0.0252	1.5205	1.5117	0.0244	0.0236
	0.3	1.5150	1.5084	0.0253	0.0249	1.5135	1.5070	0.0236	0.0233
	0.9	1.5078	1.5036	0.0245	0.0246	1.5065	1.5024	0.0229	0.0230
	1.5	1.5006	1.4988	0.0238	0.0243	1.4999	1.4978	0.0223	0.0227
	2.1	1.4954	1.4939	0.0232	0.0240	1.4937	1.4932	0.0217	0.0224

(c)

p = 0.90		Prior1				Prior2			
n	c = q	$\hat{\theta}_{LE}$	$\hat{\theta}_{GE}$	MSE1	MSE2	$\hat{\theta}_{LE}$	$\hat{\theta}_{GE}$	MSE1	MSE2
20	-1.5	1.6630	1.6093	0.1509	0.1171	1.6361	1.5887	0.1131	0.0887
	-0.9	1.6258	1.5942	0.1337	0.1115	1.6120	1.5750	0.1006	0.0845
	-0.3	1.6095	1.5790	0.1189	0.1064	1.5887	1.5613	0.0899	0.0807
	0.3	1.5842	1.5637	0.1064	0.1016	1.5661	1.5473	0.0808	0.0772
	0.9	1.5596	1.5482	0.0959	0.0973	1.5442	1.5333	0.0733	0.0742
	1.5	1.5359	1.5326	0.0872	0.0936	1.5229	1.5192	0.0670	0.0715
	2.1	1.5172	1.5169	0.0784	0.0897	1.5046	1.5039	0.0612	0.0688
40	-1.5	1.5699	1.5467	0.0527	0.0461	1.5641	1.5421	0.0469	0.0411
	-0.9	1.5581	1.5395	0.0493	0.0447	1.5529	1.5353	0.0439	0.0400
	-0.3	1.5465	1.5222	0.0463	0.0435	1.5418	1.5283	0.0413	0.0389
	0.3	1.5350	1.5250	0.0437	0.0424	1.5310	1.5214	0.0390	0.0379
	0.9	1.5237	1.5177	0.0414	0.0414	1.5202	1.5144	0.0370	0.0371
	1.5	1.5126	1.5103	0.0393	0.0405	1.5097	1.5074	0.0353	0.0363
	2.1	1.5035	1.5029	0.0371	0.0394	1.4999	1.4994	0.0338	0.0356
60	-1.5	1.5440	1.5392	0.0351	0.0321	1.5566	1.5317	0.0379	0.0347
	-0.9	1.5364	1.5344	0.0336	0.0315	1.5390	1.5270	0.0363	0.0340
	-0.3	1.5289	1.5297	0.0322	0.0309	1.5315	1.5222	0.0348	0.0334
	0.3	1.5215	1.5249	0.0310	0.0303	1.5240	1.5174	0.0335	0.0328
	0.9	1.5142	1.5201	0.0298	0.0299	1.5167	1.5126	0.0323	0.0323
	1.5	1.5069	1.5153	0.0289	0.0294	1.5094	1.5078	0.0312	0.0318
	2.1	1.4998	1.5091	0.0281	0.0289	1.5034	1.5029	0.0299	0.0314

Table 2. Average values of the Bayes estimators of λ and the corresponding MSEs

(a)

p = 0.50		Prior1				Prior2			
n	c = q	$\hat{\lambda}_{LE}$	$\hat{\lambda}_{GE}$	MSE1	MSE2	$\hat{\lambda}_{LE}$	$\hat{\lambda}_{GE}$	MSE1	MSE2
20	-1.5	1.2358	1.1192	0.5405	0.2020	1.1737	1.1006	0.1854	0.1153
	-0.9	1.1692	1.0846	0.3502	0.1839	1.1315	1.0730	0.1448	0.1054
	-0.3	1.1137	1.0497	0.2180	0.1689	1.0945	1.0453	0.1174	0.0973
	0.3	1.0690	1.0148	0.1655	0.1570	1.0615	1.0177	0.0982	0.0908
	0.9	1.0307	0.9796	0.1367	0.1482	1.0315	0.9902	0.0847	0.0860

	1.5	0.9966	0.9441	0.1181	0.1425	1.0042	0.9626	0.0750	0.0829
	2.1	0.9626	0.9087	0.1124	0.1374	0.9769	0.9349	0.0652	0.0803
40	-1.5	1.0853	1.0517	0.0818	0.0663	1.0793	1.0495	0.0649	0.0533
	-0.9	1.0657	1.0356	0.0735	0.0635	1.0619	1.0350	0.0587	0.0509
	-0.3	1.0472	1.0195	0.0667	0.0612	1.0454	1.0205	0.0535	0.0490
	0.3	1.0296	1.0033	0.0613	0.0595	1.0296	1.0060	0.0493	0.0475
	0.9	1.0129	0.9870	0.0570	0.0583	1.0145	0.9914	0.0460	0.0464
	1.5	0.9969	0.9707	0.0537	0.0577	1.0001	0.9768	0.0433	0.0458
	2.1	0.9812	0.9544	0.0511	0.0572	0.9856	0.9622	0.0419	0.0452
60	-1.5	1.0542	1.0341	0.0425	0.0375	1.0522	1.0335	0.0370	0.0327
	-0.9	1.0424	1.0235	0.0398	0.0364	1.0412	1.0237	0.0346	0.0317
	-0.3	1.0309	1.0130	0.0376	0.0355	1.0305	1.0138	0.0327	0.0309
	0.3	1.0198	1.0024	0.0357	0.0349	1.0202	1.0040	0.0310	0.0303
	0.9	1.0090	0.9918	0.0341	0.0345	1.0101	0.9941	0.0297	0.0299
	1.5	0.9985	0.9811	0.0329	0.0344	1.0003	0.9842	0.0286	0.0297
	2.1	0.9879	0.9704	0.0318	0.0343	0.9904	0.9743	0.0278	0.0295

(b)

p = 0.75		Prior1				Prior2			
n	c = q	$\hat{\lambda}_{LE}$	$\hat{\lambda}_{GE}$	MSE1	MSE2	$\hat{\lambda}_{LE}$	$\hat{\lambda}_{GE}$	MSE1	MSE2
20	-1.5	1.1280	1.0730	0.2054	0.1185	1.1102	1.0667	0.1160	0.0837
	-0.9	1.0964	1.0513	0.1540	0.1113	1.0850	1.0477	0.0977	0.0788
	-0.3	1.0681	1.0295	0.1225	0.1052	1.0618	1.0287	0.0848	0.0747
	0.3	1.0424	1.0076	0.1043	0.1001	1.0402	1.0095	0.0750	0.0713
	0.9	1.0188	0.9854	0.0917	0.0962	1.0199	0.9904	0.0675	0.0687
	1.5	0.9969	0.9632	0.0827	0.0934	1.0008	0.9712	0.0618	0.0669
	2.1	0.9749	0.9411	0.0784	0.0903	0.9817	0.9519	0.0576	0.0652
40	-1.5	1.0547	1.0348	0.0448	0.0395	1.0527	1.0341	0.0395	0.0350
	-0.9	1.0430	1.0244	0.0420	0.0384	1.0418	1.0244	0.0371	0.0339
	-0.3	1.0317	1.0140	0.0396	0.0374	1.0312	1.0147	0.0350	0.0330
	0.3	1.0207	1.0036	0.0376	0.0366	1.0209	1.0049	0.0332	0.0323
	0.9	1.0100	0.9931	0.0358	0.0361	1.0109	0.9952	0.0317	0.0318
	1.5	0.9996	0.9826	0.0345	0.0358	1.0012	0.9854	0.0305	0.0315
	2.1	0.9892	0.9721	0.0316	0.0355	0.9915	0.9756	0.0294	0.0312
60	-1.5	1.0349	1.0224	0.0283	0.0260	1.0343	1.0224	0.0240	0.0221
	-0.9	1.0275	1.0156	0.0272	0.0256	1.0272	1.0158	0.0230	0.0216
	-0.3	1.0203	1.0087	0.0262	0.0252	1.0204	1.0093	0.0221	0.0212
	0.3	1.0132	1.0018	0.0253	0.0249	1.0135	1.0026	0.0214	0.0210
	0.9	1.0062	0.9950	0.0245	0.0246	1.0069	0.9962	0.0207	0.0208
	1.5	0.9994	0.9881	0.0238	0.0243	1.0004	0.9896	0.0202	0.0207
	2.1	0.9926	0.9812	0.0232	0.0240	0.9939	0.9831	0.0197	0.0206

(c)

p = 0.90		Prior1				Prior2			
n	c = q	$\hat{\lambda}_{LE}$	$\hat{\lambda}_{GE}$	MSE1	MSE2	$\hat{\lambda}_{LE}$	$\hat{\lambda}_{GE}$	MSE1	MSE2
20	-1.5	1.0897	1.0505	0.1017	0.0810	1.0670	1.0339	0.0692	0.0568
	-0.9	1.0670	1.0321	0.0906	0.0774	1.0476	1.0175	0.0625	0.0547
	-0.3	1.0455	1.0136	0.0817	0.0746	1.0291	1.0011	0.0572	0.0531
	0.3	1.0252	0.9950	0.0746	0.0725	1.0116	0.9846	0.0530	0.0520
	0.9	1.0060	0.9763	0.0691	0.0711	0.9948	0.9680	0.0498	0.0514
	1.5	0.9877	0.9575	0.0648	0.0704	0.9788	0.9513	0.0474	0.0515
	2.1	0.9694	0.9487	0.0605	0.0697	0.9628	0.9347	0.0462	0.0514

40	-1.5	1.0333	1.0169	0.0367	0.0334	1.0341	1.0185	0.0338	0.0309
	-0.9	1.0236	1.0080	0.0350	0.0328	1.0249	1.0101	0.0323	0.0303
	-0.3	1.0141	0.9991	0.0336	0.0324	1.0159	1.0015	0.0309	0.0298
	0.3	1.0049	0.9902	0.0324	0.0321	1.0071	0.9930	0.0298	0.0295
	0.9	0.9959	0.9811	0.0314	0.0320	0.9985	0.9845	0.0289	0.0294
	1.5	0.9871	0.9721	0.0306	0.0320	0.9901	0.9760	0.0282	0.0294
	2.1	0.9783	0.9631	0.0302	0.0319	0.9817	0.9675	0.0282	0.0294
60	-1.5	1.0234	1.0128	0.0219	0.0206	1.0153	1.0049	0.0214	0.0203
	-0.9	1.0171	1.0069	0.0212	0.0203	1.0091	0.9990	0.0208	0.0201
	-0.3	1.0109	1.0009	0.0206	0.0202	1.0030	0.9930	0.0204	0.0200
	0.3	1.0048	0.9950	0.0201	0.0200	0.9970	0.9871	0.0200	0.0200
	0.9	0.9988	0.9890	0.0197	0.0200	0.9911	0.9812	0.0197	0.0200
	1.5	0.9930	0.9830	0.0194	0.0200	0.9852	0.9752	0.0194	0.0201
	2.1	0.9872	0.9769	0.0192	0.0200	0.9793	0.9682	0.0192	0.0200

Table 3. Average values of the Bayes estimators of β and the corresponding MSEs

(a)

p = 0.50		Prior1				Prior2			
n	c = q	$\hat{\beta}_{LE}$	$\hat{\beta}_{GE}$	MSE1	MSE2	$\hat{\beta}_{LE}$	$\hat{\beta}_{GE}$	MSE1	MSE2
20	-1.5	2.1531	1.2938	1.8161	0.4868	1.4253	1.1724	0.6344	0.2094
	-0.9	1.7387	1.2077	1.6395	0.3813	1.2661	1.1179	0.3386	0.1804
	-0.3	1.3524	1.1286	0.9099	0.3051	1.1655	1.0653	0.2185	0.1586
	0.3	1.1578	1.0556	0.2969	0.2516	1.0932	1.0145	0.1600	0.1433
	0.9	1.0684	0.9878	0.2025	0.2159	1.0361	0.9653	0.1274	0.1339
	1.5	1.0031	0.9245	0.1575	0.1943	0.9889	0.9177	0.1084	0.1300
	2.1	0.9379	0.8672	0.1456	0.1759	0.9419	0.8732	0.0907	0.1254
40	-1.5	1.3043	1.1422	0.5056	0.1690	1.1909	1.1047	0.1815	0.1103
	-0.9	1.2024	1.1063	0.2632	0.1497	1.1401	1.0751	0.1387	0.1002
	-0.3	1.1373	1.0717	0.1750	0.1343	1.0984	1.0463	0.1116	0.0921
	0.3	1.0898	1.0383	0.1359	0.1221	1.0628	1.0182	0.0935	0.0861
	0.9	1.0508	1.0060	0.1122	0.1129	1.0317	0.9907	0.0811	0.0818
	1.5	1.0174	0.9748	0.0967	0.1064	1.0039	0.9638	0.0726	0.0793
	2.1	0.9839	0.9436	0.0911	0.0947	0.9761	0.9369	0.0689	0.0723
60	-1.5	1.1499	1.0889	0.1390	0.0935	1.1213	1.0728	0.0991	0.0727
	-0.9	1.1143	1.0665	0.1122	0.0862	1.0930	1.0528	0.0840	0.0678
	-0.3	1.0840	1.0445	0.0942	0.0803	1.0677	1.0332	0.0730	0.0638
	0.3	1.0572	1.0232	0.0815	0.0755	1.0449	1.0139	0.0647	0.0608
	0.9	1.0334	1.0022	0.0723	0.0719	1.0240	0.9950	0.0586	0.0585
	1.5	1.0117	0.9817	0.0655	0.0692	1.0047	0.9764	0.0540	0.0571
	2.1	0.9894	0.9612	0.0612	0.0621	0.9854	0.9578	0.0502	0.0562

(b)

p = 0.75		Prior1				Prior2			
n	c = q	$\hat{\beta}_{LE}$	$\hat{\beta}_{GE}$	MSE1	MSE2	$\hat{\beta}_{LE}$	$\hat{\beta}_{GE}$	MSE1	MSE2
20	-1.5	0.4429	0.4105	0.1154	0.0525	0.3820	0.3758	0.0270	0.0231
	-0.9	0.4127	0.3799	0.0651	0.0443	0.3717	0.3543	0.0242	0.0207
	-0.3	0.3929	0.3503	0.0497	0.0385	0.3623	0.3330	0.0220	0.0193
	0.3	0.3756	0.3215	0.0421	0.0350	0.3536	0.3118	0.0202	0.0189
	0.9	0.3596	0.2935	0.0367	0.0336	0.3456	0.2908	0.0187	0.0194

	1.5	0.3459	0.2660	0.0328	0.0320	0.3381	0.2697	0.0175	0.0210
	2.1	0.3328	0.2385	0.0324	0.0319	0.3316	0.2486	0.0163	0.0209
40	-1.5	0.3702	0.3667	0.0216	0.0194	0.3599	0.3579	0.0147	0.0136
	-0.9	0.3638	0.3525	0.0200	0.0178	0.3549	0.3459	0.0139	0.0127
	-0.3	0.3578	0.3386	0.0187	0.0168	0.3502	0.3339	0.0131	0.0122
	0.3	0.3511	0.3248	0.0175	0.0161	0.3456	0.3221	0.0125	0.0119
	0.9	0.3448	0.3112	0.0165	0.0159	0.3413	0.3103	0.0120	0.0120
	1.5	0.3387	0.2977	0.0157	0.0161	0.3371	0.2986	0.0115	0.0123
	2.1	0.3316	0.2842	0.0157	0.0157	0.3319	0.2869	0.0109	0.0120
60	-1.5	0.3554	0.3541	0.0118	0.0111	0.3512	0.3503	0.0094	0.0089
	-0.9	0.3517	0.3449	0.0112	0.0105	0.3480	0.3419	0.0090	0.0085
	-0.3	0.3482	0.3358	0.0108	0.0101	0.3448	0.3337	0.0087	0.0082
	0.3	0.3447	0.3267	0.0104	0.0098	0.3418	0.3254	0.0084	0.0081
	0.9	0.3414	0.3177	0.0100	0.0098	0.3389	0.3173	0.0082	0.0081
	1.5	0.3362	0.3088	0.0097	0.0099	0.3360	0.3092	0.0079	0.0083
	2.1	0.3311	0.2999	0.0096	0.0098	0.3321	0.3011	0.0073	0.0081

(c)

p = 0.90		Prior1				Prior2			
n	c = q	$\hat{\beta}_{LE}$	$\hat{\beta}_{GE}$	MSE1	MSE2	$\hat{\beta}_{LE}$	$\hat{\beta}_{GE}$	MSE1	MSE2
20	-1.5	0.1290	0.1456	0.0096	0.0092	0.1213	0.1268	0.0031	0.0031
	-0.9	0.1253	0.1273	0.0087	0.0075	0.1197	0.1155	0.0030	0.0028
	-0.3	0.1219	0.1092	0.0080	0.0065	0.1181	0.1040	0.0029	0.0027
	0.3	0.1188	0.0910	0.0074	0.0063	0.1166	0.0922	0.0028	0.0030
	0.9	0.1160	0.0725	0.0069	0.0069	0.1142	0.0800	0.0027	0.0035
	1.5	0.1133	0.0571	0.0065	0.0074	0.1128	0.0676	0.0026	0.0043
	2.1	0.1106	0.0417	0.0062	0.0071	0.1114	0.0552	0.0024	0.0045
40	-1.5	0.1222	0.1267	0.0039	0.0039	0.1177	0.1213	0.0021	0.0021
	-0.9	0.1209	0.1174	0.0037	0.0035	0.1168	0.1142	0.0020	0.0020
	-0.3	0.1194	0.1082	0.0036	0.0033	0.1158	0.1071	0.0020	0.0019
	0.3	0.1173	0.0988	0.0035	0.0033	0.1140	0.0998	0.0019	0.0020
	0.9	0.1151	0.0893	0.0034	0.0036	0.1131	0.0924	0.0019	0.0022
	1.5	0.1130	0.0797	0.0033	0.0040	0.1123	0.0848	0.0019	0.0025
	2.1	0.1109	0.0701	0.0031	0.0038	0.1116	0.0772	0.0018	0.0023
60	-1.5	0.1170	0.1202	0.0022	0.0022	0.1198	0.1229	0.0025	0.0025
	-0.9	0.1162	0.1140	0.0021	0.0020	0.1189	0.1167	0.0025	0.0024
	-0.3	0.1152	0.1078	0.0021	0.0020	0.1179	0.1105	0.0024	0.0023
	0.3	0.1147	0.1016	0.0020	0.0020	0.1163	0.1042	0.0024	0.0023
	0.9	0.1139	0.0953	0.0020	0.0021	0.1156	0.0979	0.0023	0.0024
	1.5	0.1122	0.0889	0.0020	0.0023	0.1148	0.0915	0.0023	0.0025
	2.1	0.1105	0.0825	0.0019	0.0019	0.1139	0.0851	0.0022	0.0026

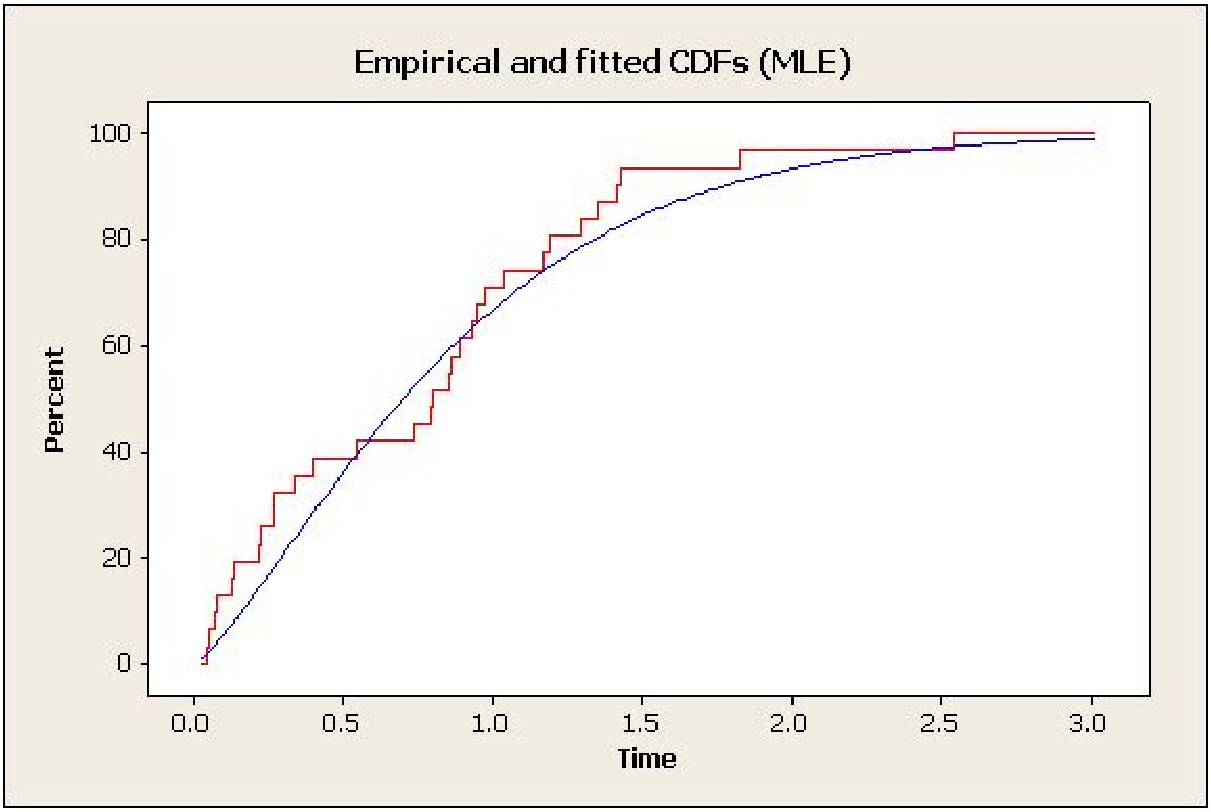
Table 4. The MLEs and the Bayes estimates under different loss functions and corresponding p-values of the Kolomogorov-Smirnov test for PBC patients in Group IV

Method	c = p	$\hat{\theta}$	$\hat{\lambda}$	$\hat{\beta}$	K-S D	p-value
MLE	-	1.2965	0.9465	0.1613	0.1450	0.5000
Bayes(SELF)	-	1.2930	0.9446	0.1665	0.1438	0.5104
Bayes(LINEXLF)	-2.5	1.3347	0.9845	0.1753	0.1449	0.5008
	-1.5	1.3177	0.9679	0.1718	0.1445	0.5045
	-0.5	1.3015	0.9525	0.1687	0.1440	0.5058
	0.5	1.2853	0.9371	0.1648	0.1437	0.5116
	1.5	1.2696	0.9227	0.1618	0.1432	0.5162

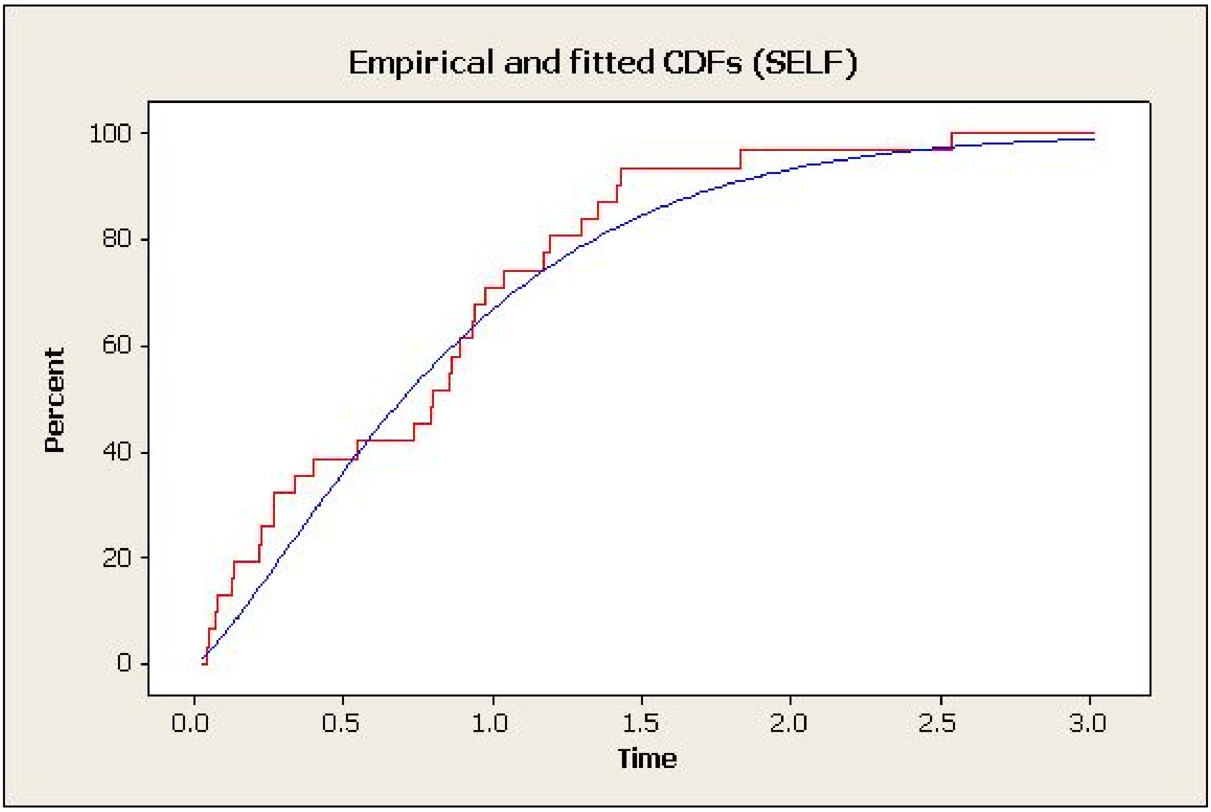
	2.5	1.2548	0.9092	0.1589	0.1428	0.5196
Bayes(GELF)	-2.5	1.3115	0.9683	0.1943	0.1401	0.5448
	-1.5	1.2993	0.9525	0.1757	0.1426	0.5218
	-0.5	1.2882	0.9366	0.1573	0.1456	0.4944
	0.5	1.2747	0.9210	0.1390	0.1478	0.4751
	1.5	1.2623	0.9047	0.1209	0.1505	0.4518
	2.5	1.2498	0.8890	0.1032	0.1532	0.4294

5 DATA ANALYSIS

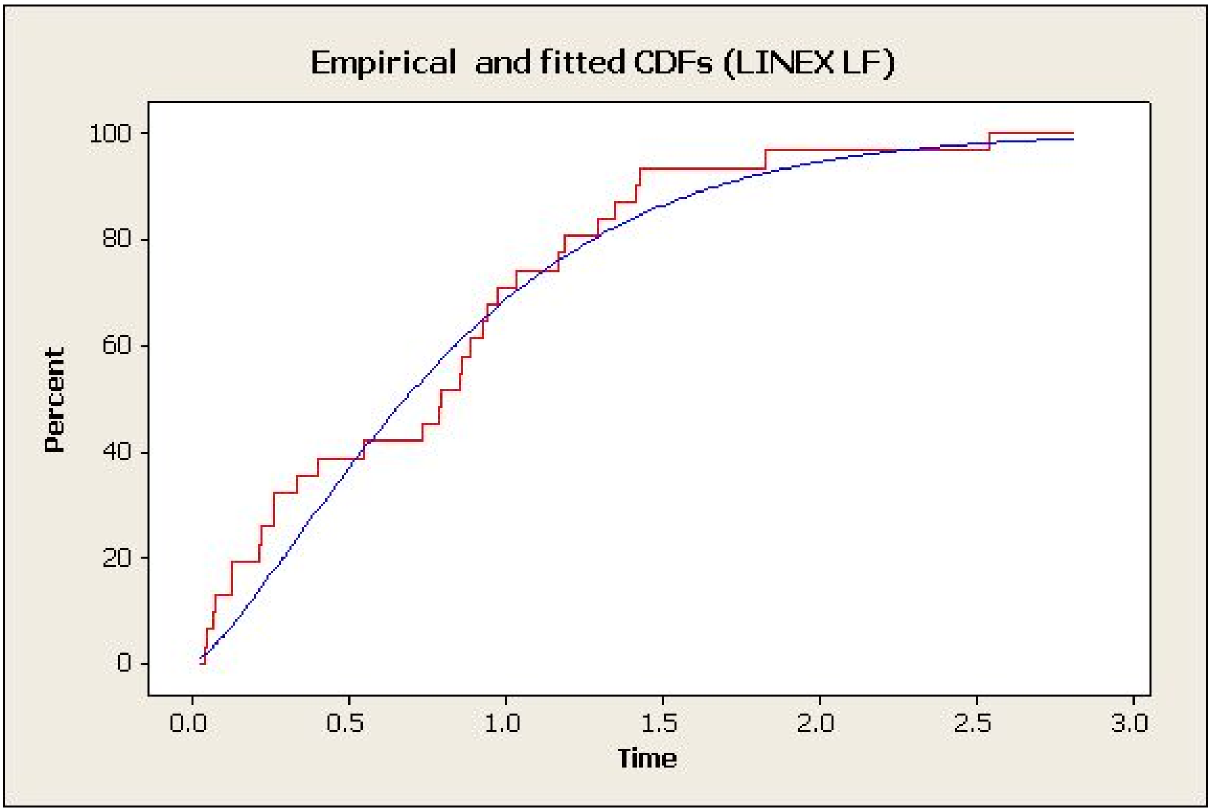
In this section, we analyze a real data set from Fleming and Harrington (1991). The data belongs to Group IV of the Primary Biliary Cirrhosis (PBC) liver study conducted by Mayo Clinic. The event of interest is time to death of PBC Patients. The data on the survival times (in days) of 36 patients who had the highest category of bilirubin are: 400, 77, 859, 71, 1037, 1427, 733, 334, 41, 51, 549, 1170, 890, 1413, 853, 216, 1882^+, 1067^+, 131, 223, 1827, 2540, 1297, 264, 797, 930, 1329^+, 264, 1350, 1191, 130, 943, 974, 790, 1765^+, 1320^+. The observations with '+' indicate censored times. For computational ease, each observation is divided by 1000. Since we do not have any prior information about the unknown parameters, we use noninformative priors with all the hyperparameters equal to zero, that is, $a_1 = b_1 = a_2 = b_2 = a_3 = b_3 = 0$, for Bayes estimates. For



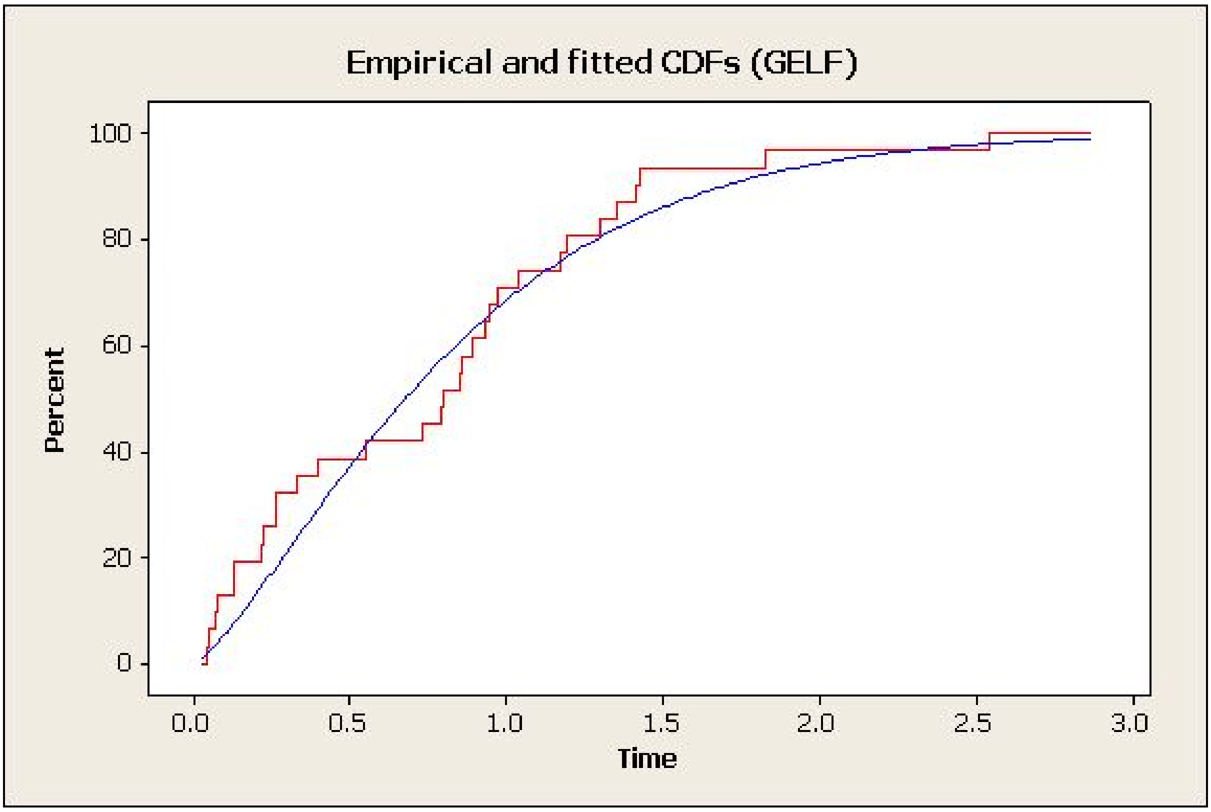
(a)


Empirical and fitted CDFs (SELF)
Percent
100
80
60
40
20
0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Time

(b)


Empirical and fitted CDFs (LINEX LF)
Percent
100
80
60
40
20
0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Time

(c)



(d)

Figure 1. Empirical and fitted CDFs using four different methods of estimation: (a) MLE, (b) Bayes (SELF), (c) Bayes (LINEXLF, c = 2.5), and (d) Bayes (GELF, q = -2.5).

different values of LFPs, we compute the Bayes estimates of θ, λ, and β under LINEX and GE loss functions. For comparison purposes, we also calculate the Bayes estimates under SE loss function and maximum likelihood estimates. To test the goodness of fit of the model to the data, we compute the Kolomogorov-Smirnov D statistics and the associated p-values for MLEs and Bayes estimates under three different loss functions. The results are given in Table 4. From these results, it can be seen that MLEs and Bayes estimates under the SE loss function are very close to each other and they can be obtained from the Bayes estimates under LINEX and GE loss functions for some choices of LFPs. We also plot the empirical and fitted CDFs using these different methods of estimation in Figure 1.

6 CONCLUSION

We consider the Bayesian estimation in the proportional hazards model of random censorship for Weibull distribution under LINEX and GE loss functions using informative and noninformative priors on the scale and the shape parameters. The Bayes estimates of parameters are obtained using the MCMC Gibbs sampling scheme. From the simulation study, it is observed that as the sample size increases, the MSEs of Bayes estimators under both the loss functions decrease. This rate of decrease in MSEs is higher for informative priors as compared to noninformative priors. This means that if one has relevant information about the parameters, it is better to use informative priors than noninformative priors. It is also noted that the rate of decrease in MSEs is significantly higher for small to moderate sample sizes as compared with moderate to large sample sizes. Therefore, it is economically better to use moderate sample sizes than large sample sizes. It is further observed that the proportion of censored observations has no significant effect on the performance of the estimators. The appropriate range for loss function parameters *c* and *q* is not the same for Bayes estimators of scale and shape parameters. However, in their respective suitable ranges, the performance of Bayes estimators under LINEX and GE loss functions is approximately equal in terms of corresponding MSEs. The results indicate that the Bayes estimators under LINEX and GE loss functions can be used effectively with the appropriate choices of LFPs. A real data analysis is performed to compare the Bayes estimators under LINEX and GE loss functions with the MLEs and Bayes estimators under the SE loss function. The performance of these different estimators is judged by the corresponding Kolomogorov-Smirnov D statistics. It is observed that the Bayes estimators under LINEX and GE loss functions with some choice of LFPs perform better than the MLEs and the Bayes estimators under the SE loss function. In this paper, we assumed the independent gamma priors on the scale and the shape parameters of the PH model; however, any other log-concave distribution, such as log-normal or Weibull etc., can also be taken as priors on these parameters.

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9 APPENDIX A

The marginal distribution of θ given data is

$$g\left(\theta\middle|\left(y,d\right)\right) \propto \theta^{n-1} e^{\theta\sum_{i=1}^{n}\ln y_i}\left(\sum_{i=1}^{n} y_i^{\theta}\right)^{-n} \qquad \text{(A)}$$

and

$$\ln\left(g(\theta)\right) = c + \left(n-1\right)\ln\theta + \theta\sum_{i=1}^{n}\ln y_i - n\ln\left(\sum_{i=1}^{n} y_i^{\theta}\right)$$

$$\frac{d}{d\theta}\ln\left(g(\theta)\right) = \frac{n-1}{\theta} + \sum_{i=1}^{n}\ln y_i - n\frac{\sum_{i=1}^{n} y_i^{\theta}\ln y_i}{\sum_{i=1}^{n} y_i^{\theta}}$$

$$\frac{d^2}{d\theta^2}\ln\left(g(\theta)\right) = \frac{-\left(n-1\right)}{\theta^2} - \frac{n}{\left(\sum_{i=1}^{n} y_i^{\theta}\right)^2}\left\{\left(\sum_{i=1}^{n} y_i^{\theta}\right)\sum_{i=1}^{n} y_i^{\theta}\left(\ln y_i\right)^2 - \left(\sum_{i=1}^{n} y_i^{\theta}\ln y_i\right)^2\right\}$$

Since

$$\left(\sum_{i=1}^{n} y_i^{\theta}\right)\sum_{i=1}^{n} y_i^{\theta}\left(\ln y_i\right)^2 - \left(\sum_{i=1}^{n} y_i^{\theta}\ln y_i\right)^2 = \sum_{i=1}^{n}\sum_{i\le j=1}^{n} y_i^{\theta} y_j^{\theta}\left(\ln y_i - \ln y_j\right)^2 \ge 0$$

Therefore

$$\frac{d^2}{d\theta^2}\ln\left(g(\theta)\right) = \frac{-\left(n-1\right)}{\theta^2} - \frac{n}{\left(\sum_{i=1}^{n} y_i^{\theta}\right)^2}\left\{\sum_{i=1}^{n}\sum_{i\le j=1}^{n} y_i^{\theta} y_j^{\theta}\left(\ln y_i - \ln y_j\right)^2\right\} < 0$$

Thus the function in (A) is log-concave.

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